

General Mathematics and CPS II

Exercise 19

May 2, 2012

1. Handout, Exercise 3.
2. Suppose a smooth periodic function $u: [0, 2\pi) \rightarrow \mathbb{C}$ is sampled at the N equidistant points $x_j = jh$ with $h = 2\pi/N$ and $j = 0, \dots, N-1$. We set $u_j = u(x_j)$, take the discrete Fourier transform

$$\tilde{u}_k = \sum_{j=0}^{N-1} e^{-ikx_j} u_j,$$

and define the reconstruction of u via

$$v(x) = \sum_{k=-N/2}^{N/2-1} e^{ikx} \tilde{u}_k.$$

Show that

$$\hat{v}_k = \begin{cases} \tilde{u}_k & \text{for } k \in -N/2, \dots, N/2 - 1, \\ 0 & \text{otherwise.} \end{cases}$$