General Mathematics and CPS II

Exercise 10

March 7, 2012

Note: This exercise is due March 21, 2012. This material will *not* be appear on the midterm exam.

1. Newton's second law of mechanics for a particle of mass m situated at position $x(t)$ moving with velocity $v(t)$ and subject to a force $F(x(t))$ can be written

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v\,,$$
$$m\,\frac{\mathrm{d}v}{\mathrm{d}t} = F(x(t))\,.$$

   Use the chain rule of calculus to show that the particle satisfies the same equation with t replaced by the reversed time $r = -t$ and v replaced by $-v$.

2. Show that in a time-discrete, reversible, system with a finite number of states any orbit must return to its initial state after a finite number of steps.

3. In the Kac ring model, N sites are placed around a circle. The sites are populated with B black balls and $W = N - B$ white balls at random. Moreover, n markers are placed on the edges between the sites at random; the number of black balls just before a marked edge is denoted b, the number of white balls just before a marked edge is w.

   Let μ denote the probability that an edge has a marker on it. Explain why

$$\mu = \frac{n}{N} = \frac{b}{B} = \frac{w}{W}\,.$$

4. Show that

$$\sum_{k=0}^{n} \binom{n}{k} k\,x^k\,y^{n-k} = n\,x\,(x+y)^{n-1}$$

   and

$$\sum_{k=0}^{n} \binom{n}{k} k^2\,x^k\,y^{n-k} = n\,x\,(x+y)^{n-1} + n\,(n-1)\,x^2\,(x+y)^{n-2}\,.$$

   Hint: Differentiate the binomial theorem.