

# Introduction to Partial Differential Equations

## Homework 5

due March 17, 2011

1. Finish the proof of the mean value formula for the heat equation by showing that

$$\iint_{E(0,0;1)} \frac{|y|^2}{s^2} dy ds = 4,$$

where

$$E(x, t; r) = \{(y, s) : \Phi(x - y, t - s) \geq r^{-n}\}$$

denotes the heat ball “centered” at  $(x, t)$ .

*Hint:* Use polar coordinates in space, and an appropriate change of variables in time. The remaining one-dimensional integral is MATHEMATICA-integrable. You can also use that

$$\begin{aligned} \int_0^\infty t^{\lambda+1} e^{-\lambda t} dt &= \frac{\Gamma(\lambda + 2)}{\lambda^{2+\lambda}}, \\ \Gamma(x + 1) &= x \Gamma(x), \\ \alpha(n) &= \frac{\pi^{n/2}}{\Gamma(\frac{n}{2} + 1)}. \end{aligned}$$

2. Evans, p. 87 problem 12
3. Evans, p. 87 problem 13