

Introduction to Partial Differential Equations

Homework 4

due March 10, 2011

1. Evans, p. 86 problem 6.
2. Let U be open and bounded with a C^1 boundary. For every $v \in C^2(\bar{U})$, set

$$J[v] = \int_U \left(\frac{1}{2} |Dv|^2 - f v \right) dx - \int_{\partial U} g v dS.$$

Assume throughout that $u \in C^2(\bar{U})$. Prove that the following two statements are equivalent.

- (i) u solves the so-called *Neumann problem*

$$\begin{aligned} -\Delta u &= f && \text{in } U, \\ \nu \cdot Du &= g && \text{on } \partial U. \end{aligned}$$

- (ii) u minimizes J , i.e.

$$J[u] \leq J[w]$$

for every $w \in C^2(\bar{U})$.

3. Evans, p. 87 problem 10.
4. Evans, p. 87 problem 11.