Introduction to Partial Differential Equations

Homework 1

due February 10, 2011

1. Evans, p. 85 problem 1
2. Evans, p. 85 problem 2
3. In the proof of the theorem on the solution of Poisson's equation we have used that

   $$D \int_{\mathbb{R}^n} \Phi(y)\, f(x-y)\, dy = \int_{\mathbb{R}^n} \Phi(y)\, Df(x-y)\, dy\,.$$

   State why and precisely under which assumptions this manipulation is permitted.
4. Consider a function of one complex variable $w \colon \mathbb{C} \to \mathbb{C}$ on an open connected subset of the complex plane, and write $w = w(z)$ with $w = u + \mathrm{i}v$ and $z = x + \mathrm{i}y$. The function w is called *(complex) differentiable* or *holomorphic* if u and v have continuous first partial derivatives with respect to x and y that satisfy the so-called *Cauchy–Riemann equations*

   $$u_x = v_y$$
   $$u_y = -v_x$$

   It is known that holomorphic functions are infinitely differentiable.

   Show that the real and imaginary parts of a holomorphic function are harmonic.