General Mathematics and ACM II

Exercise 19

May 6, 2011

1. Handout, Exercise 3.

2. Let $u\colon [0, 2\pi] \to \mathbb{R}$ be defined via the Fourier series

$$u(x) = \sum_{k \in \mathbb{Z}} \hat{u}_k \, \mathrm{e}^{\mathrm{i}kx} \, .$$

   Show that its derivative $w(x) = u'(x)$, assuming it exists, has Fourier coefficients

$$\hat{w}_k = \mathrm{i}k \, \hat{u}_k \, . \tag{*}$$

3. The previous question suggests a way to differentiate a function $u\colon [0, 2\pi] \to \mathbb{R}$ numerically: First, sample u on an equidistant grid. Second, compute the Fourier coefficients of its trigonometric interpolant v as described in class. Third, use (*) on the Fourier coefficients $\hat{v}_k$. Finally, take the inverse DFT to obtain the derivative sampled on the grid.

   The problem is that any standard software library DFT assumes that $j = 0, \ldots, N-1$ and $k = 0, \ldots, N-1$ and will store the arrys in precisely this order. Trigonometric interpolation, on the other hand, requires that $k = -N/2, \ldots, N/2-1$. With which factor, then, must the ith element, with respect to the order imposed by the software, of the vector $\hat{\boldsymbol{v}}$ be muliplied so that differentiation is computed correctly?