General Mathematics and ACM II

Exercise 11

March 23, 2011

1. Show that

$$\sum_{k=0}^{n} \binom{n}{k} k\, x^k\, y^{n-k} = n\, x\, (x+y)^{n-1}$$

and

$$\sum_{k=0}^{n} \binom{n}{k} k^2\, x^k\, y^{n-k} = n\, x\, (x+y)^{n-1} + n\,(n-1)\, x^2\, (x+y)^{n-2}\,.$$

Hint: Differentiate the binomial theorem.

2. Consider an ensemble of Kac rings with N sites where each site is colored black independently with probability ν. Show that

$$\langle \Delta \rangle = (2\nu - 1)\, N$$

and

$$\text{Var}[\Delta] = 4\,\nu\,(1-\nu)\, N\,.$$

Hint: Use the result of the previous question.

Remark: This is in fact a direct consequence of a well known result on the variance of the binomial distribution which has a rather elegant proof in terms of *moment generating functions*. These are usually introduced in a first course on probability theory. You will likely encounter them this semester in Engineering and Science Mathematics 2A/B.