General Mathematics and ACM II

Exercise 7

February 25, 2011

1. Use the matrix form of the equation for a reflection (see handout) to show that the composition of reflections about parallel lines is a translation $\Pi_{\boldsymbol{v}}$. Find an expression for the translation vector $\boldsymbol{v}$.

2. Let G be a group and let $a, b \in G$. Show that $(ab)^{-1} = b^{-1}\,a^{-1}$.

3. Let G be a finite group (i.e., a group with a finite number of elements), and let $a \in G$. Show that there exists some $n \in \mathbb{N}$ such that $a^n = e$. (Where a^n is understood as letting the group operation act between n copies of a.)

4. *(If not submitted on Friday.)*

   (Ivanov, p. 39.) Recall that the symmetry group of a subset A of the plane is defined as
   $$\mathrm{Sym}(A) = \{F \text{ motion} \colon F(A) = A\}\,.$$
   Prove that such a set of motions is indeed a group.