

Perspectives of Mathematics II

Quiz 1

March 1, 2010

Each problem is worth 5 points.

1. Let $\mathcal{V} = \mathbb{R}^2$ and let A be the non-symmetric matrix

$$A = \begin{pmatrix} \frac{1}{2} & 1 \\ 0 & \frac{1}{2} \end{pmatrix}.$$

Find the maxima and minima of

$$f(\mathbf{v}) = \mathbf{v}^T A \mathbf{v}$$

subject to $\|\mathbf{v}\| = 1$.

2. Let $\mathcal{V} = \{u \in C^2([-1, 1]) : u(-1) = u(1) = 0\}$. Find the minimum of

$$f(u) = \int_{-1}^1 u'(x)^2 dx$$

on \mathcal{V} subject to the constraint $g(u) = 0$ with

$$g(u) = \int_{-1}^1 u(x) dx - 2.$$

3. You have a set of experimentally obtained data points $(x_1, y_1), \dots, (x_n, y_n)$ of the plane and suspect that they might be related by a power law of the type

$$y = cx^\alpha.$$

How would you analyze this data and obtain the exponent α ?

(Note: a qualitative description is sufficient.)