# Complex Analysis

## Homework 2

### Due in class Wednesday, September 29, 2021

1. Let $\gamma$ be a piecewise smooth curve in $\mathbb{C}$ and let $f\colon \mathbb{C} \to \mathbb{C}$ be continuous. Show that
$$\int_\gamma f(z)\,\mathrm{d}z = -\int_{\gamma^-} f(z)\,\mathrm{d}z$$
where $\gamma^-$ denotes the curve $\gamma$ with reversed orientation.

2. Let $\gamma$ be piecewise smooth simple closed curve in $\mathbb{C}$ encircling the origin. Compute
$$\int_\gamma z^n\,\mathrm{d}z$$
for every $n \in \mathbb{Z}$.

3. Show that
$$\int_0^\infty \sin(x^2)\,\mathrm{d}x = \int_0^\infty \cos(x^2)\,\mathrm{d}x = \frac{\sqrt{2\pi}}{4}\,.$$
*Hint:* Integrate $\mathrm{e}^{-z^2}$ along the sector of the disk of radius $R$ centered at the origin that lies in the first octant of the complex plane, then let $R \to \infty$. (This is Chapter 2, Exercise 1 from Stein & Shakarchi who have a sketch of this contour in the book.)

4. Show that
$$\int_0^\infty \frac{\sin x}{x}\,\mathrm{d}x = \frac{\pi}{2}\,.$$
*Hint:* Integrate along the indented semicircle contour as in the example from class (or Stein & Shakarchi, Chapter 2, Exercise 2).

5. Let $\Omega \subset \mathbb{C}$ be open. Let $f$ be holomorphic on $\Omega$ except perhaps at a single point $w$. Show that if $f$ is bounded near $w$, then
$$\int_\gamma f(z)\,\mathrm{d}z = 0$$
for any closed piecewise smooth curve $\gamma \subset \Omega$.