# Complex Analysis

## Homework 10

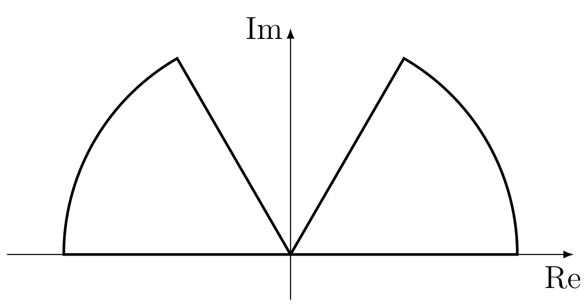
## Due in class Thursday, December 2, 2021

1. Show that
$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}\,.$$
*Note:* The proof in Stein & Sharkarchi goes via Euler's reflection formula, which is interesting in itself, but lengthy. There are much shorter proofs that are easy to find.

2. Use Laplace's method to find the leading term in the asymptotic behavior of
$$\int_{-1}^{1} \mathrm{e}^{-s\,\cosh x}\,\mathrm{d}x$$
as $s \to \infty$.

3. Fill in the details of the argument sketched in class that the Airy function
$$\mathrm{Ai}(s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}(\frac{1}{3}x^3 + sx)}\,\mathrm{d}x$$
extends to an entire function on $\mathbb{C}$.
*Hint:* As in class, shift the contour of integration to the line
$$L_\delta = \{x + \mathrm{i}\delta \colon x \in \mathbb{R}\}\,.$$

4. Show that
$$\int_{-\infty}^{\infty} x\,\mathrm{e}^{\mathrm{i}sx^3}\,\mathrm{d}x = \frac{\mathrm{i}}{\sqrt{3}}\,\Gamma(\tfrac{2}{3})\,s^{-2/3}\,.$$
*Hint:* Use contour integration along the contour




where the sectors have the opening angle $\pi/3$.