

Complex Analysis

Mock Midterm Exam

October 30, 2021

1. (a) Find all solutions to the equation

$$z^5 = 1 - i.$$

- (b) List all possible values for i^i . Which value corresponds to the principal branch of the logarithm?

(5+5)

2. What is the radius of convergence of the power series of

$$f(z) = \frac{(z-1)^2}{z^2-1}$$

about the point $z_0 = 1 + i$?

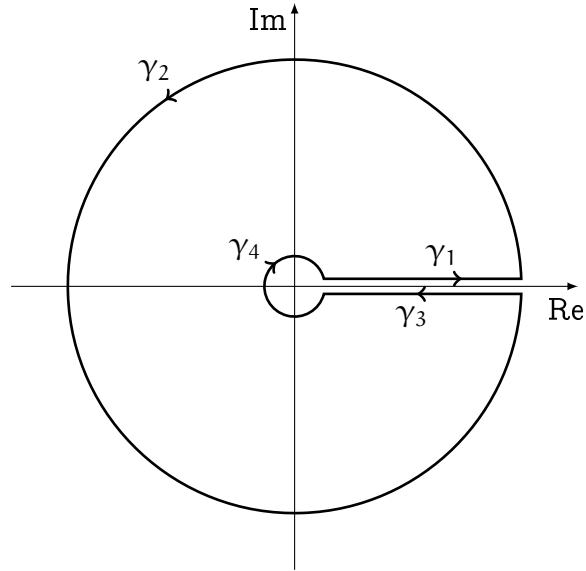
(5)

3. Integrate the square root function

$$f(z) = \sqrt{z}$$

around the unit circle $C = \{z: |z| = 1\}$ with standard orientation in two different ways:

- (a) by parameterizing the unit circle in the standard way,
(b) by integrating around a contour as sketched below.



(5+10)

4. Find

$$\int_C \frac{1}{z^2(z+1)^2} dz$$

where $C = \{z: |z| = \frac{1}{2}\}$ with standard orientation. (10)

5. Suppose $f(z)$ is entire and suppose there exist $c > 0$ and $n \in \mathbb{N}$ such that

$$|f(z)| \leq c|z|^n$$

for all $z \in \mathbb{C}$. Prove that f is a polynomial.

Hint: Consider $f^{(m)}$ for $m > n$. (10)