

Calculus and Elements of Linear Algebra I

Homework 5

Due on Moodle, Monday, October 12, 2020

1. Use implicit differentiation to find an equation of the tangent line to the graph of the given equation at the given point.

(a) $x^2 y = x + 2$ at point $(2, 1)$

(b) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 5$ at point $(8, 1)$

2. An airplane flying horizontally at a height of 5 km with a speed of 300 m/s passes directly above an observer. What is the rate of increase of distance to the observer 30 s later?
3. A balloon is filled at a rate of 100π cm³ per second. At what rate is the radius of the balloon increasing when the radius is 10 cm?
4. Show that

$$\frac{d \arcsin x}{dx} = \frac{1}{\sqrt{1-x^2}}.$$

(The function $y = \arcsin x$ is the inverse function of $x = \sin y$.)

5. Find all critical points (points where $f'(x) = 0$) for the following functions, and characterize whether they correspond to a local minimum, a local maximum, or neither.
 - (a) $f(x) = x^3 - 3x + 3$
 - (b) $f(x) = x^3 + 3x + 3$
 - (c) $g(t) = \cos(\omega t)$ with $\omega \neq 0$