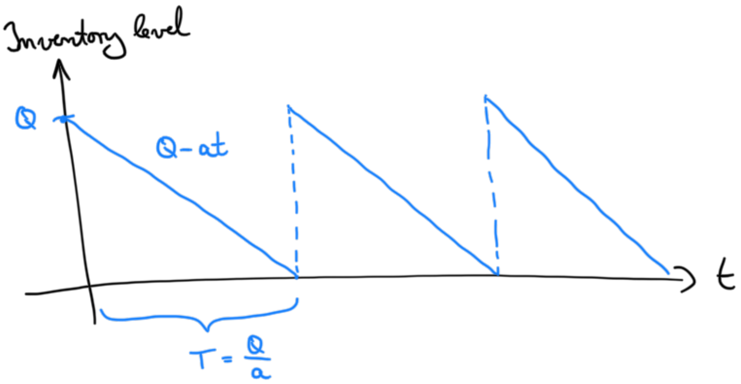
Inventory Theory

Assumptions:

- Cost of ordering:
  - K : setup cost per order
  - c : unit cost.
- Holding cost:
  - h: cost per unit per time in inventory
- withdrawal:
  - a : constant withdrawal rate $\left(\frac{\text{units}}{\text{time}}\right)$
- continuous review
- no planned shortages

→ Economic order quantity model (EOQ)

Note: It's clear that under these assumptions, new order arrives exactly when inventory is empty.



Q: what is the optimal order quantity Q^* ?

Step 1: Cost per cycle:

$$C_{cycle} = \underbrace{K + cQ}_{\text{order costs}} + \underbrace{h \overset{\uparrow\ \text{average inventory}}{\frac{Q}{2}} \overset{\text{period}}{T}}_{\text{holding cost}}$$

Step 2: Cost per time:

$$C = \frac{C_{cycle}}{T} = \frac{K + cQ}{T} + h\frac{Q}{2} = \frac{aK}{Q} + ac + \frac{1}{2}hQ$$

To find the minimum:

$$\frac{dC}{dQ} = 0 \quad \text{, here:} \quad -\frac{aK}{Q_*^2} + \frac{1}{2}h = 0$$

$$\Rightarrow \quad Q_* = \sqrt{\frac{2aK}{h}}$$

$$\text{optimal cycle time} \quad T^* = \frac{Q_*}{a} = \sqrt{\frac{2K}{ha}}$$

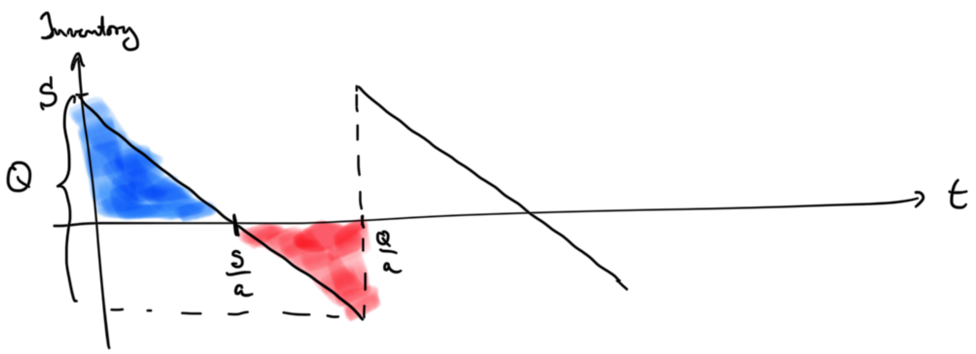
Example: $K = 12\,000$ \$ $\quad \Rightarrow \quad Q^* = 25\,298$

$h = 0.30$ per month $\quad T^* = 3.2$ months

$a = 8000$ units per month

EOQ with planned shortages:

as before, but inventory can be empty for part of a cycle at a penalty of p per unit per time; withdrawals that cannot be fulfilled will be postponed and processed when the new batch arrives.



$$C_{cycle} = \underbrace{K + cQ}_{\text{order cost}} + \underbrace{h\,\tfrac{1}{2}S \cdot \tfrac{S}{a}}_{\text{holding cost}} + \underbrace{p\,\frac{Q-S}{2}\,\frac{Q-S}{a}}_{\text{penalty}}$$

$$C = \frac{C_{cycle}}{T} = \frac{Ka}{Q} + ca + \frac{1}{2}h\frac{S^2}{Q} + \frac{1}{2}p\frac{(Q-S)^2}{Q}$$

$$\left(1 - \frac{S}{Q}\right)(Q-S)$$

$$\frac{\partial C}{\partial S} = \frac{hS}{Q} - p\,\frac{Q-S}{Q} = 0 \qquad (A)$$

$$\frac{\partial C}{\partial Q} = -\frac{Ka}{Q^2} - \frac{1}{2}h\frac{S^2}{Q^2} + \frac{1}{2}p\left[\frac{S}{Q^2}(Q-S) + \frac{Q-S}{Q}\right] = 0 \qquad (B)$$

$$\overline{\partial Q} \qquad Q^2$$

$$(A) \Rightarrow \quad hS = p(Q-S) \Rightarrow (h+p)S = pQ \Rightarrow S = \frac{p}{h+p}Q$$

$$(B) \Rightarrow \quad -\frac{Ka}{Q^2} - \frac{1}{2}h\cancel{\frac{S^2}{Q^2}} + \frac{1}{2}h\left[\cancel{\frac{S^2}{Q^2}} + \frac{S}{Q}\right] = 0$$

$$\Rightarrow \frac{Ka}{Q} = \frac{1}{2}hS = \frac{1}{2}h\frac{p}{h+p}Q$$

$$\Rightarrow Ka = \frac{1}{2}h\frac{p}{h+p}Q^2$$

$$\Rightarrow \boxed{Q^* = \sqrt{\frac{2Ka}{h}}\sqrt{\frac{h+p}{p}} \qquad S^* = \sqrt{\frac{2Ka}{h}}\sqrt{\frac{p}{h+p}}}$$

Periodic review models

r_i : demand in period i

K : setup cost

c : cost of order/production per unit (NOT relevant)

h : holding cost per item at end of period.

Example: Small airplane manufacturer

$r_1 = 3 \quad r_2 = 2, \quad r_3 = 3, \quad r_4 = 2$

$K = 2$ M\$

$h = 0.2$ M\$

Note: In optimal schedule, produce only when inventory is empty.