Decision Trees

Goferbroke Oil company:

Payoffs in k$

Alternatives:	States: Oil	Dry
Drill	700	-100
Sell	90	90
Prop. (Prior) p	$\frac{1}{4}$	$\frac{3}{4}$

Task: Maximize expected payoff

$$\mathbb{E}[\text{Drill}] = 700p - 100(1-p)$$

$$\mathbb{E}[\text{Sell}] = 90 = 90p + 90 \cdot (1-p)$$

Q: For what probability p is it worth drilling?

Cut-over: $700p - 100(1-p) = 90$

$\Rightarrow 800p = 190$

$\Rightarrow p = \frac{190}{800} \approx 0.24$

(So $p = \frac{1}{4}$ is just in the range where drilling is preferred.)

Exploration: Can make decision to spend 30 k$ on exploration, where

From past data: $P(\text{Favorable} \mid \text{Oil}) = 0.6$ $\quad P(\text{Unfavorable} \mid \text{Oil}) = 0.4$

$P(\text{Favorable} \mid \text{Dry}) = 0.2$ $\quad P(\text{Unfavorable} \mid \text{Dry}) = 0.8$

Q: $P(\text{Oil} \mid \text{Favorable})$? Bayes' rule!!!

$$P(O \mid F) = \frac{P(F \mid O)\,P(O)}{P(F)}$$

Recall:

$P(A \cap B) = P(A \mid B)\,P(B)$

$\shortparallel$

$P(B \cap A) = P(B \mid A)\,P(A)$

So we need to find $P(F)$:

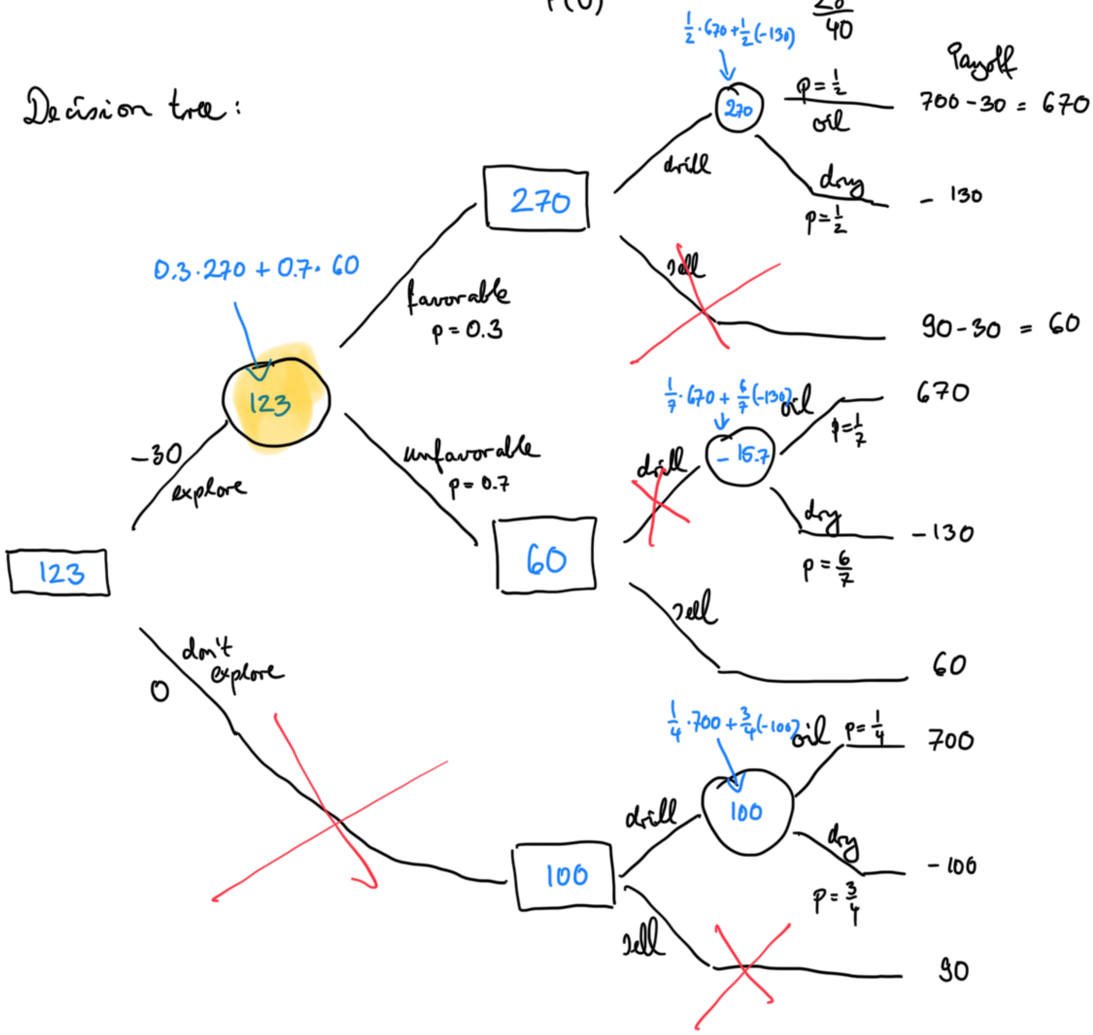
$$P(F) = P(F \mid O)\,P(O) + P(F \mid D)\,P(D)$$

$$= 0.6\,\tfrac{1}{4} + 0.2\,\tfrac{3}{4} = \frac{6}{40} + \frac{6}{40} = \frac{12}{40} = \frac{3}{10}$$

$$\Rightarrow P(\theta \mid F) = \frac{0.6 \cdot \frac{1}{4}}{\frac{12}{40}} = \frac{\frac{6}{40}}{\frac{12}{40}} = \frac{1}{2}$$

$$P(\theta \mid U) = \frac{P(U \mid \theta)\, P(\theta)}{P(U)} = \frac{0.4 \cdot \frac{1}{4}}{\frac{28}{40}} = \frac{4}{28} = \frac{1}{7}$$

Decision tree:



<u>Result:</u>

- explore first
- if unfavorable, sell
- if favorable, drill

Overall: expected profit is 123000 \$.

<u>Expected value of experimentation:</u> EVE:

EVE = expected profit with experimentation − expected profit without experimentation

$= 153 - 100 = 53$

Expected value of perfect information : EPVI

$$EVPI = \text{expected profit if state is perfectly known} - \text{expected profit without information}$$

$$= \frac{1}{4} \cdot 700 + \frac{3}{4} 90 - 100$$

$$= 242.5 - 100$$

$$= 142.5$$