Probabilistic dynamic programming

Example: "Hit-and-miss Manufacturing Co."

$p = \frac{1}{2}$: product meets specifications

For each batch: \$300 set-up cost
\$100 cost per item

Time for 3 batches, if no good product is produced, penalty of \$1600

State variable s: # of items still to be produced, here 0 or 1

Objective f: expected total cost

decision variables: x_1, x_2, x_3 : # of item to be produced in stage $i = 1, .., 3$

$K(x)$ is the set-up cost of a batch, so
$K(x) = 0$ for $x = 0$
$K(x) = 3$ for $x > 0$

n = 3:

$$f_3(1, x_3) = K(x_3) + x_3 + \left(\frac{1}{2}\right)^{x_3} \cdot 16$$

s	$x_3=0$	$x_3=1$	$x_3=2$	$x_3=3$	$x_3=4$	$x_3=5$	f_3^*	x_3^*
0	0	—	—	—			0	0
1	16	3+1+8 = 12	3+2+4 = 9	3+3+2 = 8	3+4+1 = 8	3+5+$\frac{1}{2}$ = 8$\frac{1}{2}$	8	3 or 4

n = 2:

$$f_2(1, x_2) = K(x_2) + x_2 + \left(\frac{1}{2}\right)^{x_2} \cdot f_3^*(1)$$

s	$x_2=0$	$x_2=1$	$x_2=2$	$x_2=3$	$x_2=4$		f_2^*	x_2^*
0	0	—	—	—	—	—	0	0
1	8	3+1+$\frac{1}{2}\cdot 8$ = 8	3+2+$\frac{1}{4}\cdot 8$ = 7	3+3+1 = 7	3+4+$\frac{1}{2}$ = 7$\frac{1}{2}$		7	2 or 3

n = 1:

$$f_1(1, x_1) = K(x_1) + x_1 + \left(\frac{1}{2}\right)^{x_1} \cdot f_2^*(1)$$

s	$x_1=0$	$x_1=1$	$x_1=2$	$x_1=3$	$x_1=4$	f_1^*	x_1^*
1	7	3+1+$\frac{1}{2}$7 = 7$\frac{1}{2}$	3+2+$\frac{1}{4}$7 = 6$\frac{3}{4}$	3+3+$\frac{7}{8}$ = 6$\frac{7}{8}$	-----	6$\frac{3}{4}$	2

Conclusion:

2 items in first batch, if not successful, 2 or 3 items in second batch, if not successful, third batch with 3 or 4 items.