Two questions about project mangement:

1. What is the minimal time to completion, and what is the critical path, assuming that each activity i takes a known fixed time T_i

- decision variables t_i, starting time of activity i

minimize t_{FINISH}

subject to $t_j \geq t_i + T_i$ if j depends on i

$t_{START} = 0$

$t_i \geq 0$

2. Suppose that the time for completion is prescribed, and shorter than the critical path in 1.

Assume activity i can be "crashed" by x_i units of time, second set of decision variables:

x_i units of time saved on activity i

T_i regular time for completion

R_i: max time that can be saved

c_i: cost of saving one unit of time

LP: minimize $\sum_i c_i x_i$

subject to $x_i \leq R_i$ for each activity i

$t_j \geq t_i + (T_i - x_i)$ "

$t_i \geq 0$; $x_i \geq 0$

Spring 2016 last two questions: too difficult

Fall 2017 #1: max $z = 3x_1 + 2x_2 + x_3$

subject to $x_1 + x_2 + x_3 \leq 2$

$3x_1 + x_2 \leq 1$

$x_1, x_2, x_3 \geq 0$

min $-z = -3x_1 - 2x_2 - x_3$

$x_1 + x_2 + x_3 + s_1 = 2$

$3x_1 + x_2 + s_2 = 1$

$s_1, s_2 \geq 0$

x_1	x_2	x_3	s_1	s_2	
1	1	1	1	0	2
3	1	0	0	1	1
−3	−2	−1	0	0	0

Start with s_1, s_2 basic, this is clearly feasible.

For convenience, take x_2 as entering (permitted, but not the standard choice)

$R1 - R2 \to R1$	−2	0	1	1	−1	1
	3	1	0	0	1	1
$2R2 + R3 \to R3$	3	0	−1	0	2	2

Now x_3 must be entering:

	−2	0	1	1	−1	1
	3	1	0	0	1	1
$R1 + R3 \to R3$	1	0	0	1	1	3

This is the final tableau: $x_1 = 0$ (non-basic!)

$x_2 = 1$

$x_3 = 1$

$s_1 = s_2 = 0$ (non-basic!)

$-z = -3$, $\boxed{z = 3}$