Operations Research

Final Exam

December 21, 2017

1. Use the graphical method to maximize

$$z = x + y$$

subject to

$$x^2 + y^2 \le 20\,,$$
$$x\,y \le 8\,,$$
$$x, y \ge 0\,.$$

(20)

2. The attached Pyomo notebook shows a solution to the "Diet Problem".

   (a) State a concise formulation of the problem *in words*. (No need to repeat the given numerical values of the data.)

   (b) Describe the meaning of the dual variables in words.

   (c) Which nutrient is supplied at more than its minimal requirement? Explain your answer.

   (d) Suggest an additional constraint (or additional constraints) to ensure that the resulting diet is more varied (without changing given data of the problem). State the idea in words and then write out the corresponding lines of Pyomo code.

(5+5+5+5)

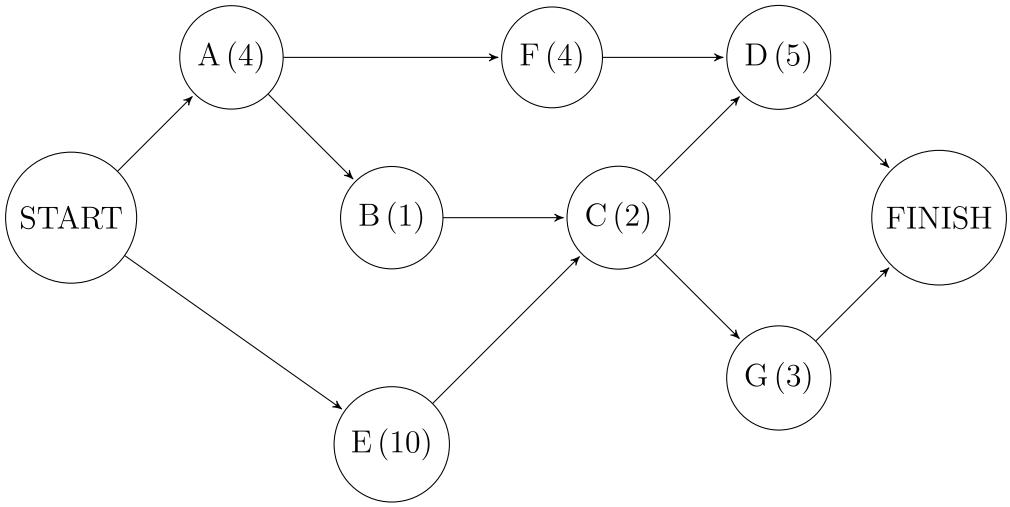
3. You have decided to take advantage of your holidays and of the recent heavy snowfall to try snowboarding. You hire the equipment at the ski resort. The shop has three different offers: one day hiring for 25 €, two days 40 €, and four days for 60 €.

   The weather forecast for the next 8 days is given in the following table where a 1 indicates that conditions are good for snowboarding and a 0 indicates that conditions are bad for snowboarding.

Day	1	2	3	4	5	6	7	8
Weather	1	0	1	1	0	1	0	1

Use dynamic programming to find out when and for how long to hire a snowboard in order to minimize the total expense assuming you go snowboarding if and only if the weather is good. (20)

4. A project consists of activities A, …, G which have the following dependencies:



The time (in days) that each step takes is indicated in parentheses. Each day that the project is being worked on (independent of how many activities are done in parallel) costs 1 000 €. Furthermore, special machinery must be rented from the beginning of task A up to the end of task G at a cost of 5 000 € per day.

(a) Formulate a linear program that minimizes the total cost of the project. You do not need to solve it, but you are required to state a complete mathematical formulation.

(b) In this case, the problem is simple enough to analyze it "by hand". Find the optimal solution.

(10+10)

5. Consider the Goferbroke Co. problem discussed in class with the following modified payoff table:

	State of Nature	
Alternative	**Oil**	**Dry**
Drill	60	-15
Sell	10	10
Prior Probability	1/3	2/3

The Goferbroke Co. has to decide whether to drill for oil or to sell the land.

(a) Compute the expected payoffs for both alternatives. What decision should be taken based on a risk-neutral analysis?

(b) A company offers exploration which returns the true state of nature in 2/3 of cases and gives a false result in 1/3 of cases. Compute the probability of obtaining a favorable exploration result (i.e., one that indicates presence of oil), and the probability of finding a non-favorable exploration result.

(c) How much should you be willing to pay for these services?

(d) How do you decide upon drilling or selling in response to the exploration result?

(5+5+5+5)

In [1]:
```
from pyomo.environ import *
from pyomo.opt import *
opt = solvers.SolverFactory("glpk")
```

In [2]:
```
F = ['Broccoli', 'Milk', 'Oranges']
N = ['Calcium', 'Vitamin C', 'Water']

c = {'Broccoli':38, 'Milk':10, 'Oranges':27}
r = {'Calcium':1000, 'Vitamin C':90, 'Water':3700}

a = {('Broccoli','Calcium'):47,
     ('Broccoli','Vitamin C'):89,
     ('Broccoli','Water'):91,
     ('Milk','Calcium'):276,
     ('Milk','Vitamin C'):0,
     ('Milk','Water'):87,
     ('Oranges','Calcium'):40,
     ('Oranges','Vitamin C'):53,
     ('Oranges','Water'):87}

model = ConcreteModel()
model.x = Var(F, within=NonNegativeReals)
```

In [3]:
```
def nutrition_rule (model, n):
    return sum(a[i,n]*model.x[i] for i in F) >= r[n]

model.c = Constraint(N, rule=nutrition_rule)

model.z = Objective(expr = sum(c[i]*model.x[i] for i in F),
                    sense=minimize)
```

In [4]:
```
model.dual = Suffix(direction=Suffix.IMPORT)
results = opt.solve(model)
```

In [5]:
```
model.x.get_values()
```

Out[5]:
```
{'Broccoli': 1.01123595505618, 'Milk': 41.4710060699987, 'Oran
ges': 0.0}
```

In [6]:
```
model.z.expr()
```

Out[6]:
```
453.1370269921219
```

In [7]:
```
for n in N:
    print("Shadow price of", n, "is", model.dual[model.c[n]])
```

```
Shadow price of Calcium is 0.0
Shadow price of Vitamin C is 0.309440785225365
Shadow price of Water is 0.114942528735632
```