Operations Research

Homework 3

Due in class Wednesday, September 28, 2016

Note: All Pyomo homework is due Thursday, September 29, in class.

1. Reconsider Problem 1 from Homework Set 1: *Minimize*

$$z = 8\,x + 12\,y$$

subject to

$$\begin{gathered} 5\,x + 2\,y \geq 20\,, \\ 4\,x + 3\,y \geq 24\,, \\ y \geq 2\,, \\ x, y \geq 0\,. \end{gathered}$$

Introduce slack variables to write this linear programming problem in the standard form: *Minimize*

$$z = \boldsymbol{c}^T \boldsymbol{x}$$

subject to

$$\begin{gathered} A\boldsymbol{x} = \boldsymbol{b} \\ \boldsymbol{x} \geq 0 \end{gathered}$$

where the coefficients $\boldsymbol{b}, \boldsymbol{c}$, and the decision variables $\boldsymbol{x}$ are written as column vectors, and A is a matrix of matching dimension.

2. Solve Problem 1 using the simplex method *on paper.*

3. (From HL, problem 3.4-15.) A cargo plane has three compartments for storing cargo: front, center, and back. These compartments have capacity limits on both *weight* and *space*, as summarized below:

Compartment	**Weight Capacity** [t]	**Space Capacity** [ft^3]
Front	12	7 000
Center	18	9 000
Back	10	5 000

Furthermore, the weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the airplane.

The following four cargoes have been offered for shipment on an upcoming flight as space is available:

Cargo	**Weight** [t]	**Volume** [ft^3/t]	**Profit** [$/t]
1	20	500	320
2	16	700	400
3	25	600	360
4	13	400	290

Any portion of these cargoes can be accepted. The objective is to determine how much (if any) of each cargo should be accepted and how to distribute each among the compartments to maximize the total profit for the flight.

(a) Formulate a linear programming model for this problem. (Submission in mathematical notation!)

(b) Solve this model using Pyomo to find one of its multiple optimal solutions. (Submission as printout of Ipython notebook.)