Applied Calculus

Homework 5

Due in class, November 3, 2015

1. (From MLS, p. 405.)

   The population of a flock of geese is modeled by

   $$G(t) = 2000 + 150\,\sin\left(\frac{\pi}{6}\,t\right) + 75\,\sin\left(\frac{\pi}{3}\,t\right),$$

   where t is measured in months from January 1.

   (a) Find any critical points and any inflection points of G. State the location of any local maxima, minima, and inflection points. Use this information to sketch a graph of $G(t)$ over a 2-year period.

   (b) Interpret any local maxima, minima, and inflection points over the first 2 years in biological terms.

   (c) How fast is the flock growing on May 1? On May 1, is the *rate* at which the flock is growing increasing or decreasing?

2. (From MLS, p. 406.) During an epidemic, the number of infected people $I(t)$ is approximated by

   $$I(t) = \frac{a\,\ln(bt+1)}{bt+1},$$

   where t is measured in days since the initial outbreak.

   (a) Find an expression for the time when the number of infected people starts to decline.

   (b) Suppose that you introduce a treatment for infected individuals. When treatment is added to the model, the value of b is increased. What effect does this have on the time at which the number of infected people starts to decline? Given your answer, describe (in biological terms) the population-level effect of your treatment.

   (c) Consider your answers from (a) and (b). Is there a biological/physical limitation to how large the value of b can be? (*Hint:* Try calculating the time at which the number of infected people starts to decline for a *very large* value of b.)

3. (From MLS, p. 406.) For infants less than 9 months old, the relationship between the rate of growth R (in pounds/month) and the present weight W (in pounds) is approximated by

$$R = c\,W\,(21 - W)$$

for some constant c. At what weight is the growth rate the largest?

4. (From MLS, p. 407.) Imagine that you have just been given a vacation home situated on an island 10 km off the coast. The nearest town is 60 km up the coast from the point of least distance to the island. Suppose you can travel at 100 km/h along the highway and at 20 km/h in water, at which point should you land to minimize travel time from the island to town?

5. (From MLS, p. 406.) Find the rectangle with the largest area that can be placed inside the region bounded between the curve $y = 4 - x^2$ and the horizontal axis $y = 0$.