Applied Calculus

Homework 3

Due in class, October 6, 2015

Note: Homework 2, Problems 5 and 6 are also due October 6.

1. Compute the derivative of $f(x) = x^2$ in three different ways.
   (a) Use the explicit definition of the derivative as the difference quotient
   $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\,.$$
   (b) Use the power rule.
   (c) Use the product rule.

2. Compute the derivative of the following functions.
   (a) $f(x) = 3\,x^4 + \dfrac{1}{2}\,x^2 + 32$
   (b) $f(x) = x^{3/2} + x^{-3/2}$
   (c) $f(x) = x\,\mathrm{e}^x$
   (d) $f(x) = \dfrac{e^x}{x}$
   (e) $f(x) = \sqrt{1 + x^2}$
   (f) $f(x) = \dfrac{1+x}{1-x}$
   (g) $f(x) = x\,\ln x$
   (h) $f(x) = \ln(rx + x^2)$ where r is a constant
   (i) $f(x) = \mathrm{e}^{x/k}$ where k is a constant

   (2 Points each)

3. Determine whether the following functions are differentiable. If not, state the point(s) of non-differentiability. Explain your answer.
   (a) $f(x) = |x|$

(b) $f(x) = x^{1/3}$

(c) $f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$

(d) $f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x^2 & \text{for } x \geq 0 \end{cases}$

(e) $f(x) = \dfrac{1+x}{1-x}$

(2 Points each)