Applied Calculus

Homework 1

Due in class, September 15, 2015

Note: You may express your answer in terms of unevaluated exponential or logarithmic functions so long as everything that can be simplified by hand is simplified. There is no need to use a calculator unless explicitly requested.

1. (a) Find the equation of the line through the points $(4, 5)$ and $(8, 7)$.
   (b) Find x and y intercepts for the line with equation $y = 2x - 5$.
   (c) Find the point of intersection of the lines with equations

   $$y = -x + 7 \qquad \text{and} \qquad y = 3x - 9\,.$$

   (d) Find an equation for the line parallel to the line with equation $y = 10x+5$ through the point $(0, 2)$.

2. The cost of a taxi ride is the sum of a flat call out fee and a charge for every kilometer traveled. In some city, a ride of 6 km length costs EUR 12 while a ride of 10 km length costs EUR 18. How much are the call out fee and the per-kilometer charge? How much does a trip of 15 km cost?

3. Find the equation for a quadratic through the points $(1, 15)$, $(3, 31)$, and $(-2, 6)$.

4. (From MLS, p. 67.) Solve each of the exponential equations for t.
   (a) $4\,(3^{2t}) = 12$
   (b) $6\,(0.6)^t = 10$
   (c) $4\,\mathrm{e}^{0.05t} = 8$
   (d) $3^{t^2-t} = 9$

5. (From MLS, p. 67.) Solve each of the following logarithmic equations for x.
   (a) $\ln(2x + 3) = 5$
   (b) $\log_2(\log_3 2x) = 4$

6. (From MLS, p. 67.) Salmonella bacteria grow rapidly at room temperature. If some bacteria are left on the cutting board when a chicken with salmonella is cut up and they get into a salad, the population of bacteria begins growing. Suppose that the number present in the salad after t hours is given by

$$f(t) = 200 \cdot \mathrm{e}^{rt}$$

where $r = 2\,\mathrm{h}^{-1}$.

   (a) If the salad is left at room temperature, how many bacteria are present 1 hour later?

   (b) How many bacteria were initially present in the salad?

   (c) What is the doubling time for the number of bacteria?

7. (From MLS, p. 68.) Suppose that the island area A and the number of species S (of a certain taxa) on that island are given by

$$S = \alpha\, A^{\beta}\,,$$

where α and β are constants.

   (a) What kind of function is this?

   (b) Suppose you are studying land and freshwater birds among a small cluster of islands. The first island has an area of 50 square miles, and you count 13 species of birds. the second island has an area of 125 square miles, and you count 17 species of birds. Given this information, find the values of α and β so that you have a description of how bird species count depends on island size.

   (c) If you count 24 species of birds on a nearby island, estimate the area of this island.

8. (From MLS, p. 68.) Suppose that for a certain data set, the semilog (base-10) graph is a line through the points $(2, -2)$ and $(3, -4)$. Give an equation for y as a function of x.

9. *Note:* The following problem requires the use of *Scientific Python*. It should be attempted this week, but is due only one week later to allow ironing out potential technical problems. An Ipython noteboook from the class session September 3 is available on my web page, it already contains the solution for part (a) below.

   (From MLS, p. 45.) Use the data from the Example shown in class.

   (a) Find the equation for the least-squares fit of the y data in terms of the x data.

   (b) Find the equation for the least-squares fit of the x data in terms of the y data.

   (c) Solve the equation in (b) for y in terms of x. Is this the same equation that you obtained in (a)? Explain why you think the answers were the same or different.

   (d) Compute the ρ^2 value in each case and interpret the results.