Mathematics Preparedness Self-Assessment

Jacobs University

Fall Semester 2015

Instructions: Set aside two hours of quality time where you can work undisturbed. Try to answer as many of these question as possible "closed book", i.e., without looking up any information in books or online.

Solutions will be posted on Sunday, September 6, after 18:00 together with instructions on how to self-evaluate your results, on

`http://math.jacobs-university.de/oliver/`

Problems

1. Compute
$$\frac{2}{3} - \frac{1}{6}\,.$$

2. Solve the quadratic equation
$$2x^2 + x - 1 = 0\,.$$

3. Solve
$$\frac{kT - 1}{T} = S$$
for T.

4. Solve the simultaneous equations
$$\begin{aligned} 3x + 2y &= 1\,, \\ 6x - 4y &= 3\,. \end{aligned}$$

5. Simplify the expression
$$\frac{(\mathrm{e}^b)^{1+\ln a}}{\mathrm{e}^b}\,.$$

6. Sketch the graph of the function
$$y = 3\,\cos 2x$$
including appropriate labeling of the coordinate axes.

7. Compare the functions

$$f(x) = \frac{3x}{(x-1)(x+2)} \qquad \text{and} \qquad g(x) = \frac{1+x}{1-x^2} - \frac{2}{2+x}\,.$$

Are they the same, or do they differ?

8. Find all local minima and maxima of the function

$$f(x) = x^3 - 3x + 1\,.$$

9. Consider the function

$$f(x) = x\,\mathrm{e}^{-x}\,.$$

Find its horizontal and vertical asymptotes, local minima, local maxima, and points of inflection. Identify the regions where the graph of f is concave upward or concave downward. Draw a sketch of the graph.

10. Compute

$$\int_0^{2\pi} x\,\sin x\,\mathrm{d}x\,.$$

11. Consider the Fibonacci sequence

$$a_{n+1} = a_n + a_{n-1} \qquad \text{for } n \geq 1\,,$$
$$a_0 = a_1 = 1\,.$$

Show that this sequence $\{a_n\}$ is unbounded.

12. How many roots (zeros) does the function

$$f(x) = x^7 + x^5 + x^3 + x$$

have?

13. A triangle has edges of length 3, 4, and 5. Is it a right-angled triangle?

14. Show that for arbitrary real numbers a and b,

$$a\,b \leq \tfrac{1}{2}\,a^2 + \tfrac{1}{2}\,b^2\,.$$

15. Find the indefinite integral

$$\int \sqrt{\frac{1-x}{1+x}}\,\mathrm{d}x\,.$$