Name:

Quiz 7

Recall: Error/uncertainty propagation in one variable

$$\Delta y = |f'(x)|\,\Delta x$$

and in two variables, assuming errors in x and y are *independent*,

$$\Delta z = \sqrt{\left(\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{\partial f}{\partial y}\Delta y\right)^2}.$$

1. If you measure $x = 25 \pm 2$, what should you report for $\sqrt{x}$, together with its uncertainty?

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} \Rightarrow \Delta y \approx \left|\frac{1}{2}\frac{1}{5}\right|\Delta x = \frac{1}{5}$$

$$\Rightarrow \sqrt{x} = 5 \pm \frac{1}{5}$$

2. If $y = x^3$, what is the *relative* error in y in terms of the relative error in x?

$$\Delta y \approx \left|\frac{\partial y}{\partial x}\right|\Delta x = 3x^2\,\Delta x$$

$$\Rightarrow \frac{\Delta y}{y} = \frac{3x^2}{x^3}\Delta x = 3\,\frac{\Delta x}{x}$$

3. If you measure $x = 2 \pm 2$ and $y = 5 \pm 3$, what should you report for $z = 2x + y$, together with its uncertainty?

$$\Delta z^2 \approx \left(\frac{\partial z}{\partial x}\Delta x\right)^2 + \left(\frac{\partial z}{\partial y}\Delta y\right)^2 = (2\cdot 2)^2 + (1\cdot 3)^2 = 16+9 = 25$$

$$\Rightarrow z = 9 \pm 5$$ (assuming independent uncertainties!)

1. Find the indefinite integral $\int e^{-x}\,dx$.

$$\int e^{-x}\,dx = -e^{-x} + c$$

2. Find the definite integral $\int_1^2 x^3 - x\,dx$.

$$\int_1^2 x^3 - x\,dx = \tfrac{1}{4}x^4 - \tfrac{1}{2}x^2 \Big|_1^2 = \tfrac{1}{4}(2^4 - 1^4) - \tfrac{1}{2}(2^2 - 1^2)$$

$$= 4 - \tfrac{1}{4} - 2 + \tfrac{1}{2} = 2\tfrac{1}{4}$$

3. Find the definite integral $\int_0^2 2x\,e^{-x^2}\,dx$.

$u = x^2$ $\quad du = 2x\,dx$

$$\int_0^2 2x\,e^{-x^2}\,dx = \int_0^4 e^{-u}\,du = -e^{-u}\Big|_0^4$$

$$= 1 - e^{-4}$$