Homework solutions

HW8 Q2: (a) $\int x^2\sqrt{x^3+1}\,dx$

$U = x^3+1$

$\frac{du}{dx} = 3x^2 \Rightarrow \frac{du}{3} = x^2\,dx$

$= \int \sqrt{U}\,\frac{du}{3}$

$= \frac{2}{9}U^{\frac{3}{2}} + C = \frac{2}{9}(x^3+1)^{\frac{3}{2}} + C$

(b) $\int \tan\theta\,d\theta = \int \frac{\sin\theta}{\cos\theta}\,d\theta$

$U = \cos\theta$

$\Rightarrow \frac{du}{d\theta} = -\sin\theta$

$\Rightarrow \sin\theta\,d\theta = -du$

$= \int \frac{-du}{U} = -\ln U + C$

$= -\ln\cos\theta + C$

(Note: This is valid only for $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$!)

(c) $\int t\sin t\,dt = t(-\cos t) - \int 1\cdot(-\cos t)\,dt$

$= -t\cos t + \sin t + C$

(d) $\int t^2\sin t\,dt = t^2(-\cos t) - \int 2t(-\cos t)\,dt$

$= -t^2\cos t + 2\underbrace{\int t\cos t\,dt}$

$= t\sin t - \int 1\cdot\sin t\,dt$

$= t\sin t + \cos t + C$

$= -t^2\cos t + 2t\sin t + 2\cos t + C'$

Q3: (a) $\displaystyle\int_0^1 r\sqrt{1-r^2}\,dr$ $\qquad U = 1-r^2$, $\quad du = -2r\,dr$

$$= \int_1^0 \sqrt{U}\,\frac{-du}{2}$$

$$= \frac{1}{2}\int_0^1 U^{\frac{1}{2}}\,dU = \frac{1}{3}U^{\frac{3}{2}}\Big|_0^1 = \frac{1}{3}$$

(b) $\displaystyle\int_{-\pi}^{\pi} \sin\theta\,d\theta = 0$

($\sin\theta$ is an odd function integrated over a symmetric interval. Of course, you can also find the answer as usual via the FTC ...)

(c) $\displaystyle\int_1^2 y\,(y^2+1)^{\frac{3}{2}}\,dy$ $\qquad U = y^2+1 \Rightarrow du = 2y\,dy$

$$= \frac{1}{2}\int_2^5 U^{\frac{3}{2}}\,du = \frac{1}{2}\,\frac{2}{5}\,U^{\frac{5}{2}}\Big|_2^5 = \frac{1}{5}\left(5^{\frac{5}{2}} - 2^{\frac{5}{2}}\right) = 5\sqrt{5} - \frac{4}{5}\sqrt{2}$$

(d) $\displaystyle\int_0^\infty x\,e^{-x^2}\,dx = -\frac{1}{2}\int_0^{-\infty} e^{U}\,dU$ $\qquad U = -x^2$, $\quad dU = -2x\,dx$

$$= -\frac{1}{2}e^{U}\Big|_0^{-\infty} = -\frac{1}{2}\left(0 - e^{0}\right) = \frac{1}{2}$$

Q4 (a): $\displaystyle C = \int_0^{10} R(t)\,dt = \int_0^{10} 4t\,e^{-0.05t}\,dt = \frac{4}{-0.05}\,t\,e^{-0.05t}\Big|_0^{10} - \int_0^{10}\frac{4}{-0.05}\,e^{-0.05t}\,dt$

$$= \frac{4}{-0.05}\,10\,e^{-0.5} + \frac{4}{0.05}\,\frac{1}{(-0.05)}\,e^{-0.05t}\Big|_0^{10}$$

$$= -800\,e^{-0.5} - 1600\,(e^{-0.5} - 1) \approx 144.3$$

(b) $\displaystyle C_{total} = \int_0^\infty R(t)\,dt = 0 + \int_0^\infty 80\,e^{-0.05t}\,dt = -1600\,e^{-0.05t}\Big|_0^\infty = 1600$