

Homework solutions

HW 7 Q1: $\frac{\partial y}{\partial x} = a \frac{b}{bx} = \frac{a}{x}$

$$\Rightarrow \Delta y \approx \left| \frac{\partial y}{\partial x} \right| \Delta x = a \frac{\Delta x}{x}$$

Q2: (a) $A(5760) = \frac{A_0}{2} \Rightarrow \frac{1}{2} = e^{-k \cdot 5760}$

$$\Rightarrow \ln 2 = k \cdot 5760$$

$$\Rightarrow k = \frac{\ln 2}{5760} \approx 1.2 \cdot 10^{-4}$$

(b) $0.1 = e^{-kt} \Rightarrow \ln 0.1 = -kt$

$$\Rightarrow t = \frac{\ln 0.1}{-k} = 5760 \frac{\ln 0.1}{\ln 2} \approx 19100 \text{ years}$$

(c) $r = e^{-kt} \Rightarrow \ln \frac{1}{r} = kt \Rightarrow t = \frac{\ln \frac{1}{r}}{k}$

$$\Rightarrow \Delta t \approx \left| \frac{\partial t}{\partial r} \right| \Delta r = \left| \frac{1}{k} \frac{-1}{r} \right| \Delta r = \frac{1}{k} \frac{\Delta r}{r}$$

$$= \frac{1}{k} \cdot 0.1 \approx 830 \text{ years}$$

So the age of the sample is 19100 ± 830 years.

Q3: We assume independent uncertainties (cf. class discussion & handout), so

$$(\Delta z)^2 \approx \left(\frac{\partial z}{\partial x} \Delta x \right)^2 + \left(\frac{\partial z}{\partial y} \Delta y \right)^2$$

We compute

$$\frac{\partial z}{\partial x} = \frac{y(y-x) - (-1)xy}{(y-x)^2} = \frac{y^2}{(y-x)^2}$$

$$\frac{\partial z}{\partial y} = \frac{x(y-x) - xy}{(y-x)^2} = \frac{-x^2}{(y-x)^2}$$

$$\Rightarrow \Delta z^2 \approx \left(\frac{15^2}{5^2} \right)^2 + \left(\frac{10^2}{5^2} \cdot 2 \right)^2 = 81 + 16 = 97$$

$$\Rightarrow \Delta z \approx \sqrt{97} \approx 10$$

$$z = \frac{10 \cdot 15}{5} = 30$$

$$\Rightarrow z = 30 \pm 10$$

Q4: Again, we assume independent uncertainties.

$$\begin{aligned} \Delta u^2 &\approx \left(\frac{\partial u}{\partial x} \Delta x \right)^2 + \left(\frac{\partial u}{\partial y} \Delta y \right)^2 + \left(\frac{\partial u}{\partial z} \Delta z \right)^2 \\ &= (yz \Delta x)^2 + (xz \Delta y)^2 + (xy \Delta z)^2 \end{aligned}$$

$$\Rightarrow \left(\frac{\Delta u}{u} \right)^2 \approx \left(\frac{\Delta x}{x} \right)^2 + \left(\frac{\Delta y}{y} \right)^2 + \left(\frac{\Delta z}{z} \right)^2$$

$$\Rightarrow \frac{\Delta u}{u} \approx \sqrt{\left(\frac{\Delta x}{x} \right)^2 + \left(\frac{\Delta y}{y} \right)^2 + \left(\frac{\Delta z}{z} \right)^2}$$