HW6 Q1:

$$\frac{\partial f}{\partial x} = 2x - 1$$

$$\frac{\partial f}{\partial y} = 1 + 2y$$

Critical point: $2x - 1 = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

$1 + 2y = 0 \Rightarrow y = -\frac{1}{2}$

As this is the only critical point and $f(x,y) \to \infty$ as either $x \to \infty$ or $y \to \infty$, this critical point corresponds to a minimum of the function.

At the critical point, $f_{min} = f(\frac{1}{2}, -\frac{1}{2}) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} - \frac{1}{2} + \left(-\frac{1}{2}\right)^2 = -\frac{1}{2}$

Q2b): $A = 2xy$

constraint: $g(x,y) = x^2 + y = 4$

Lagrange equations:

$$\left.\begin{aligned} \frac{\partial A}{\partial x} = \lambda \frac{\partial g}{\partial x} &\Rightarrow 2y = \lambda 2x \\ \frac{\partial A}{\partial y} = \lambda \frac{\partial g}{\partial y} &\Rightarrow 2x = \lambda \cdot 1 \end{aligned}\right\} \quad y = 2x^2$$

Plug into constraint: $2x^2 = 4 - x^2$

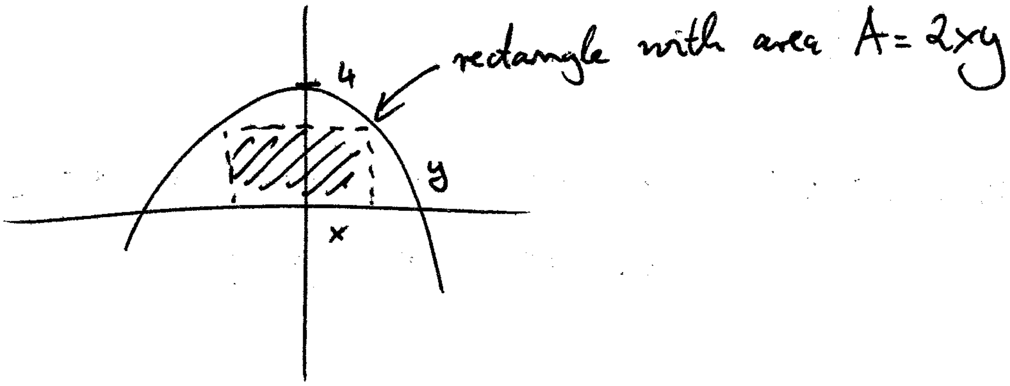
$\Rightarrow x^2 = \frac{4}{3}$

$\Rightarrow x = \frac{2}{\sqrt{3}}$ (Neg. root does not make sense here.)

$\Rightarrow y = \frac{8}{3}$

As the area is 0 when the rectangle is squeezed to a line, this candidate solution corresponds to the global maximum.

Sketch:



<u>Q3:</u> distance function $f = \text{distance}^2 = (1-x)^2 + (2-y)^2$

constraint function $g = x^2 + y^2 = 1$

Lagrange equations:

$$\left.\begin{aligned} \frac{\partial f}{\partial x} = \lambda \frac{\partial g}{\partial x} &\Rightarrow -2(1-x) = \lambda 2x \\ \frac{\partial f}{\partial y} = \lambda \frac{\partial g}{\partial y} &\Rightarrow -2(2-y) = \lambda 2y \end{aligned}\right\} \Rightarrow \frac{1-x}{2-y} = \frac{x}{y}$$

$$\Rightarrow y(1-x) = (2-y)x \Rightarrow y - xy = 2x - xy \Rightarrow y = 2x$$

$$\Rightarrow x^2 + (2x)^2 = 1 \Rightarrow x^2 = \frac{1}{5} \Rightarrow x = \pm \frac{1}{\sqrt{5}},\ y = \pm \frac{2}{\sqrt{5}}$$

$$f\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) = \left(1 - \frac{1}{\sqrt{5}}\right)^2 + \left(2 - \frac{2}{\sqrt{5}}\right)^2 = \frac{(\sqrt{5}-1)^2 + 4(\sqrt{5}-1)^2}{5} = (\sqrt{5}-1)^2$$

$$f\left(-\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right) = \ldots\ldots = (\sqrt{5}+1)^2$$

So the first critical point corresponds to the minimal distance, the second critical point corresponds to the maximal distance.