Applied Calculus

Midterm Exam II

October 15, 2015

1. Suppose that for a certain data set, the semilog graph (base-10, logarithm on the y-axis) is a line through the points $(1, -3)$ and $(2, -1)$. Give an equation for y as a function of x.

(10)

2. Compute the derivative of the following functions.

   (a) $f(x) = (1 + x)^2$

   (b) $f(x) = \dfrac{ax}{bx + c}$ where a, b, and c are constants.

   (c) $f(x) = x^2 e^{-x}$

(5+5+5)

3. The arctan function is a special function. You will find the following formula for its derivative in any table of mathematical functions:

$$(\arctan x)' = \frac{1}{1 + x^2}\,.$$

   Use this information to find the derivative of

$$f(x) = \arctan \frac{1}{x^4}\,.$$

(10)

4. Find the derivative of

$$f(x) = \frac{1}{x}$$

   by explicitly computing the limit of the difference quotient

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}\,.$$

(10)

5. Find the equation of the tangent line at $x = 0$ for the graph of the function

$$k(x) = \ln(1 + rx)$$

where r is a given constant. (10)

6. Consider the function

$$f(x) = \frac{1}{2x} + \ln 2x$$

For which values of x is the function defined? Find the vertical and horizontal asymptotes (if any), find and classify all critical points, determine where the function is concave up or concave down, find all points of inflection, and sketch the graph.

(State the explicit y values for the special points you found only where they are easy to compute.) (5+5+5+5+5)

7. You drive the 300 km distance from Berlin to Hamburg at an average speed of 100 km/h. Then you go on to Bremen, which is 100 km from Hamburg, where there is a traffic jam on the A1 so that you average speed drops down to 50 km/h on this part of your journey. What is your average speed over the entire distance? (10)