Applied Calculus

Midterm Exam I

September 24, 2015

1. (a) Find the equation of the line through the points $(3, -2)$ and $(-1, 1)$.

   (b) Find the equation of the quadratic polynomial $y = a\,x^2 + b\,x + c$ through the points $(0, 0)$, $(3, 0)$, and $(1, 1)$.

   (5+10)

2. Solve the following equations for x.

   (a) $2^x = 4$

   (b) $\log_{10}(6 \log_2 x) = 3$

   (c) $e^{\frac{2 \ln x^s}{s}} = 4$

   (5+5+5)

3. Suppose that for a certain data set, the doubly logarithmic (base-10) graph is a line through points $(1, 3)$ and $(2, -4)$. Give an equation for y as a function of x. (10)

4. The number of bacteria in milk grows exponentially, at least for some time. At bottling time, it is known that there are 10^6 bacteria per bottle, the next day, there are three times as many. The milk can be consumed with up to 10^9 bacteria per bottle.

   (a) Determine the shelf-life of bottled milk under these assumptions.

   (b) You are investigating the growth of bacteria in a sample of food and want to plot the number of bacteria vs. time. Which scaling function will you use on each of the coordinate axes, and why?

   (10+5)

5. Compute the following limits.

(a) $\lim_{r \to 10} \frac{\log_{10} r}{r}$

(b) $\lim_{r \to 2} \frac{r^2 - 4}{r - 2}$

(c) $\lim_{r \to \infty} \frac{e^r + r}{e^r}$

(5+5+5)

6. Determine whether $g(x)$ is continuous. If $g(x)$ has a discontinuity, state the type of discontinuity (removable discontinuity, jump discontinuity, vertical asymptote, or other).

   (a) $g(x) = \frac{1}{x}$

   (b) $g(x) = x \ln x^2$

   (c) $g(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ x^2 & \text{for } x > 0 \end{cases}$

(5+5+5)

7. An elevator is driven by a motor which is either off or moves it with the constant speed of 1 m/s up or down.

   The elevator is initially at the bottom of a building. At time $t = 20\,\mathrm{s}$, it visits the third floor at height $h = 12\,\mathrm{m}$, then at $t = 60\,\mathrm{s}$ the first floor at height $h = 4\,\mathrm{m}$.

   (a) Is the height function $h(t)$ continuous? Why or why not?

   (b) Draw a possible height function $h(t)$ onto the graph paper provided. Label the coordinate axes carefully.
   (Note: The answer is not unique!)

(5+10)