Derivatives Lab

Session 15

November 6, 2012

Let $X = X(t)$ be an Itō process, i.e., a solution of the stochastic differential equation

$$\mathrm{d}X = u(X,t)\,\mathrm{d}t + v(X,t)\,\mathrm{d}W\,,$$

interpreted in the sense of the Itō stochastic integral. Let $f(X,t)$ be twice continuously differentiable. Then the stochastic chain rule, also known as the Itō formula, reads

$$\mathrm{d}f(X,t) = \left(\frac{\partial f(X,t)}{\partial t} + u(X,t)\,\frac{\partial f(X,t)}{\partial X} + \frac{1}{2}\,v(X,t)^2\,\frac{\partial^2 f(X,t)}{\partial X^2}\right)\mathrm{d}t + v(X,t)\,\frac{\partial f(X,t)}{\partial X}\,\mathrm{d}W\,.$$

Verify the Itō formula numerically for the example when $X(t)$ is geometric Brownian motion with $\mu = 0.2$ and $\sigma = 2.0$, and where

$$f(X,t) = t\,\sqrt{X}\,.$$

Hint: You have to compare direct evaluation of this expression with a numerical solution of the stochastic differential equation which you obtain from the Itō formula.