Derivatives Lab

Session 19

November 22, 2011

1. Recall that the explicit form of the finite difference approximation to the Black–Scholes equation reads

$$\frac{V_n^m - V_n^{m-1}}{\Delta t} + \frac{\sigma^2}{2}\,\frac{V_{n-1}^m - 2\,V_n^m + V_{n+1}^m}{\Delta X^2} + \left(r - \frac{\sigma^2}{2}\right)\frac{V_{n+1}^m - V_{n-1}^m}{2\,\Delta X} - r\,V_n^m = 0\,.$$

   Further, recall the definition $S = \exp(X)$.

   Write a code which uses the explicit finite difference scheme to price a European call option. (What are the boundary conditions?)

2. Show that the explicit code becomes unstable unless the time step Δt is rather small.

3. Modify your code to use the implicit finite difference scheme

$$\frac{V_n^{m+1} - V_n^m}{\Delta t} + \frac{\sigma^2}{2}\,\frac{V_{n-1}^m - 2\,V_n^m + V_{n+1}^m}{\Delta X^2} + \left(r - \frac{\sigma^2}{2}\right)\frac{V_{n+1}^m - V_{n-1}^m}{2\,\Delta X} - r\,V_n^m = 0\,.$$

   Show that it is stable even when the time step Δt is large.

4. Demonstrate the order of convergence of the implicit finite difference method with the same number of meshpoints in t and X variable.