$$\int \frac{1}{x}(1+\ln x)^5\,dx = \ln x + \frac{5}{2}(\ln x)^2 + \frac{10}{3}(\ln x)^3 + \frac{5}{2}(\ln x)^4 + (\ln x)^5 + \frac{1}{6}(\ln x)^6$$

that looks considerably less appealing than the hand result $\frac{1}{6}(1+\ln x)^6$ of Example 1. Is the relationship between the two obvious? See Problem 54.

9.2 PROBLEMS

Evaluate the integrals in Problems 1 through 30.

1. $\int (2-3x)^4\,dx$

2. $\int \frac{1}{(1+2x)^2}\,dx$

3. $\int x^2\sqrt{2x^3-4}\,dx$

4. $\int \frac{5t}{5+2t^2}\,dt$

5. $\int \frac{2x}{\sqrt[3]{2x^2+3}}\,dx$

6. $\int x\sec^2 x^2\,dx$

7. $\int \frac{\cot\sqrt{y}\,\csc\sqrt{y}}{\sqrt{y}}\,dy$

8. $\int \sin\pi(2x+1)\,dx$

9. $\int (1+\sin\theta)^5\cos\theta\,d\theta$

10. $\int \frac{\sin 2x}{4+\cos 2x}\,dx$

11. $\int e^{-\cot x}\csc^2 x\,dx$

12. $\int \frac{e^{\sqrt{x+4}}}{\sqrt{x+4}}\,dx$

13. $\int \frac{(\ln t)^{10}}{t}\,dt$

14. $\int \frac{t}{\sqrt{1-9t^2}}\,dt$

15. $\int \frac{1}{\sqrt{1-9t^2}}\,dt$

16. $\int \frac{e^{2x}}{1+e^{2x}}\,dx$

17. $\int \frac{e^{2x}}{1+e^{4x}}\,dx$

18. $\int \frac{e^{\arctan x}}{1+x^2}\,dx$

19. $\int \frac{3x}{\sqrt{1-x^4}}\,dx$

20. $\int \sin^3 2x\cos 2x\,dx$

21. $\int \tan^4 3x\sec^2 3x\,dx$

22. $\int \frac{1}{1+4t^2}\,dt$

23. $\int \frac{\cos\theta}{1+\sin^2\theta}\,d\theta$

24. $\int \frac{\sec^2\theta}{1+\tan\theta}\,d\theta$

25. $\int \frac{(1+\sqrt{x})^4}{\sqrt{x}}\,dx$

26. $\int t^{-1/3}\sqrt{t^{2/3}-1}\,dt$

27. $\int \frac{1}{(1+t^2)\arctan t}\,dt$

28. $\int \frac{\sec 2x\tan 2x}{(1+\sec 2x)^{3/2}}\,dx$

29. $\int \frac{1}{\sqrt{e^{2x}-1}}\,dx$

30. $\int \frac{x}{\sqrt{\exp(2x^2)-1}}\,dx$

In Problems 31 through 35, evaluate the given integral by making the indicated substitution.

31. $\int x^2\sqrt{x-2}\,dx$; $u = x-2$

32. $\int \frac{x^2}{\sqrt{x+3}}\,dx$; $u = x+3$

33. $\int \frac{x}{\sqrt{2x+3}}\,dx$; $u = 2x+3$

34. $\int x\sqrt[3]{x-1}\,dx$; $u = x-1$

35. $\int \frac{x}{\sqrt[3]{x+1}}\,dx$; $u = x+1$

In Problems 36 through 50, evaluate the given integral. First make a substitution that transforms it into a standard form. The standard forms with the given formula numbers are inside the back cover of this book. If a computer algebra system is available, compare and reconcile (if necessary) the result found using the integral table formula with a machine result.

36. $\int \frac{1}{100+9x^2}\,dx$; formula (17)

37. $\int \frac{1}{100-9x^2}\,dx$; formula (18)

38. $\int \sqrt{9-4x^2}\,dx$; formula (54)

39. $\int \sqrt{4+9x^2}\,dx$; formula (44)

40. $\int \frac{1}{\sqrt{16x^2+9}}\,dx$; formula (45)

41. $\int \frac{x^2}{\sqrt{16x^2+9}}\,dx$; formula (49)

42. $\int \frac{x^2}{\sqrt{25+16x^2}}\,dx$; formula (49)

43. $\int x^2\sqrt{25-16x^2}\,dx$; formula (57)

44. $\int x\sqrt{4-x^4}\,dx$; formula (54)

45. $\int e^x\sqrt{9+e^{2x}}\,dx$; formula (44)

46. $\int \frac{\cos x}{(\sin^2 x)\sqrt{1+\sin^2 x}}\,dx$; formula (50)

47. $\int \frac{\sqrt{x^4-1}}{x}\,dx$; formula (47)

48. $\int \frac{e^{3x}}{\sqrt{25+16e^{2x}}}\,dx$; formula (49)

49. $\int \frac{(\ln x)^2}{x}\sqrt{1+(\ln x)^2}\,dx$; formula (48)

50. $\int x^8\sqrt{4x^6-1}\,dx$; formula (48)

EXAMPLE 7 With $n = 4$ in Eq. (5) we get

$$\int \sec^4 x\,dx = \tfrac{1}{3}\sec^2 x \tan x + \tfrac{2}{3}\int \sec^2 x\,dx = \tfrac{1}{3}\sec^2 x \tan x + \tfrac{2}{3}\tan x + C, \tag{8}$$

and with $n = 5$ we get

$$\int \sec^5 x\,dx = \tfrac{1}{4}\sec^3 x \tan x + \tfrac{3}{4}\int \sec^3 x\,dx = \tfrac{1}{4}\sec^3 x \tan x + \tfrac{3}{8}\sec x \tan x + \tfrac{3}{8}\ln|\sec x + \tan x| + C, \tag{9}$$

using Eq. (6) in the last step. ▩

9.3 PROBLEMS

Use integration by parts to compute the integrals in Problems 1 through 34.

1. $\int xe^{2x}\,dx$
2. $\int x^2e^{2x}\,dx$
3. $\int t\sin t\,dt$
4. $\int t^2\sin t\,dt$
5. $\int x\cos 3x\,dx$
6. $\int x\ln x\,dx$
7. $\int x^3\ln x\,dx$
8. $\int e^{3z}\cos 3z\,dz$
9. $\int \arctan x\,dx$
10. $\int \frac{\ln x}{x^2}\,dx$
11. $\int \sqrt{y}\ln y\,dy$
12. $\int x\sec^2 x\,dx$
13. $\int (\ln t)^2\,dt$
14. $\int t(\ln t)^2\,dt$
15. $\int x\sqrt{x+3}\,dx$
16. $\int x^3\sqrt{1-x^2}\,dx$
17. $\int x^5\sqrt{x^3+1}\,dx$
18. $\int \sin^2\theta\,d\theta$
19. $\int \csc^3\theta\,d\theta$
20. $\int \sin(\ln t)\,dt$
21. $\int x^2\arctan x\,dx$
22. $\int \ln(1+x^2)\,dx$
23. $\int \sec^{-1}\sqrt{x}\,dx$
24. $\int x\tan^{-1}\sqrt{x}\,dx$
25. $\int \tan^{-1}\sqrt{x}\,dx$
26. $\int x^2\cos 4x\,dx$
27. $\int x\csc^2 x\,dx$
28. $\int x\arctan x\,dx$
29. $\int x^3\cos x^2\,dx$
30. $\int e^{-3x}\sin 4x\,dx$
31. $\int \frac{\ln x}{x\sqrt{x}}\,dx$
32. $\int \frac{x^7}{(1+x^4)^{3/2}}\,dx$
33. $\int x\cosh x\,dx$
34. $\int e^x\cosh x\,dx$

In Problems 35 through 38, first make a substitution of the form $t = x^k$ and then integrate by parts.

35. $\int x^3\sin x^2\,dx$
36. $\int x^7\cos x^4\,dx$
37. $\int \exp(-\sqrt{x})\,dx$
38. $\int x^2\sin x^{3/2}\,dx$

In Problems 39 through 42, use the method of cylindrical shells to calculate the volume of the solid obtained by revolving the region R around the y-axis.

39. R is bounded below by the x-axis and above by the curve $y = \cos x$, $-\pi/2 \leqq x \leqq \pi/2$.
40. R is bounded below by the x-axis and above by the curve $y = \sin x$, $0 \leqq x \leqq \pi$.
41. R is bounded below by the x-axis, on the right by the line $x = e$, and above by the curve $y = \ln x$.
42. R is bounded below by the x-axis, on the left by the y-axis, on the right by the line $x = 1$, and above by the curve $y = e^{-x}$.

In Problems 43 through 45, first estimate graphically or numerically the points of intersection of the two given curves, then approximate the volume of the solid that is generated when the region bounded by these two curves is revolved around the y-axis.

43. $y = x^2$ and $y = \cos x$
44. $y = 10x - x^2$ and $y = e^x - 1$
45. $y = x^2 - 2x$ and $y = \ln(x+1)$
46. Use integration by parts to evaluate

$$\int 2x\arctan x\,dx,$$

with $dv = 2x\,dx$, but let $v = x^2 + 1$ rather than $v = x^2$. Is there a reason why v should not be chosen in this way?

47. Use integration by parts to evaluate $\int xe^x \cos x\,dx$.

48. Use integration by parts to evaluate $\int \sin 3x \cos x\,dx$.

Derive the reduction formulas given in Problems 49 through 54. Throughout, n denotes a positive integer with an appropriate side condition (such as $n \geqq 1$ or $n \geqq 2$).

49. $\displaystyle\int x^n e^x\,dx = x^n e^x - n\int x^{n-1}e^x\,dx$

50. $\displaystyle\int x^n e^{-x^2}\,dx = -\frac{1}{2}x^{n-1}e^{-x^2} + \frac{n-1}{2}\int x^{n-2}e^{-x^2}\,dx$

51. $\displaystyle\int (\ln x)^n\,dx = x(\ln x)^n - n\int(\ln x)^{n-1}\,dx$

52. $\displaystyle\int x^n \cos x\,dx = x^n \sin x - n\int x^{n-1}\sin x\,dx$

53. $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx$

54. $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1}x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx$

Use appropriate reduction formulas from the preceding list to evaluate the integrals in Problems 55 through 57.

55. $\displaystyle\int_0^1 x^3 e^x\,dx$ **56.** $\displaystyle\int_0^1 x^5 e^{-x^2}\,dx$

57. $\displaystyle\int_1^e (\ln x)^3\,dx$

58. Apply the reduction formula in Problem 53 to show that for each positive integer n,

$$\int_0^{\pi/2} \sin^{2n}x\,dx = \frac{\pi}{2}\cdot\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdots\frac{2n-1}{2n}$$

and

$$\int_0^{\pi/2} \sin^{2n+1}x\,dx = \frac{2}{3}\cdot\frac{4}{5}\cdot\frac{6}{7}\cdot\frac{8}{9}\cdots\frac{2n}{2n+1}.$$

59. Derive the formula

$$\int \ln(x+10)\,dx = (x+10)\ln(x+10) - x + C$$

in three different ways: (a) by substituting $u = x + 10$ and applying the result of Example 1; (b) by integrating by parts with $u = \ln(x+10)$ and $dv = dx$, noting that

$$\frac{x}{x+10} = 1 - \frac{10}{x+10};$$

and (c) by integrating by parts with $u = \ln(x+10)$ and $dv = dx$, but with $v = x + 10$.

60. Derive the formula

$$\int x^3 \tan^{-1}x\,dx = \tfrac{1}{4}(x^4-1)\tan^{-1}x - \tfrac{1}{12}x^3 + \tfrac{1}{4}x + C$$

by integrating by parts with $u = \tan^{-1}x$ and $v = \frac{1}{4}(x^4 - 1)$.

61. Let $J_n = \int_0^1 x^n e^{-x}\,dx$ for each integer $n \geqq 0$. (a) Show that

$$J_0 = 1 - \frac{1}{e} \quad\text{and that}\quad J_n = nJ_{n-1} - \frac{1}{e}$$

for $n \geqq 1$. (b) Deduce by mathematical induction that

$$J_n = n! - \frac{n!}{e}\sum_{k=0}^{n}\frac{1}{k!}$$

for each integer $n \geqq 0$. (c) Explain why $J_n \to 0$ as $n \to +\infty$. (d) Conclude that

$$e = \lim_{n\to\infty}\sum_{k=0}^{n}\frac{1}{k!}.$$

62. Let m and n be positive integers. Derive the reduction formula

$$\int x^m(\ln x)^n\,dx = \frac{x^{m+1}}{m+1}(\ln x)^n - \frac{n}{m+1}\int x^m(\ln x)^{n-1}\,dx.$$

63. An advertisement for a symbolic algebra program claims that an engineer worked for three weeks on the integral

$$\int (k\ln x - 2x^3 + 3x^2 + b)^4\,dx,$$

which deals with turbulence in an aerospace application. The advertisement said that the engineer never got the same answer twice during the three weeks. Explain how you could use the reduction formula of Problem 62 to find the engineer's integral (but don't actually do it). Can you see any reason why it should have taken three weeks?

64. Figure 9.3.4 shows the region bounded by the x-axis and the graph of $y = \frac{1}{2}x^2\sin x$, $0 \leqq x \leqq \pi$. Use Formulas (42) and (43) (inside the back cover)—which are derived by integration by parts—to find (a) the area of this region; (b) the volume obtained by revolving this region around the y-axis.

65. The top shown in Fig. 9.3.5 has the shape of the solid obtained by revolving the region of Problem 64 around the x-axis. Find the volume of this top.

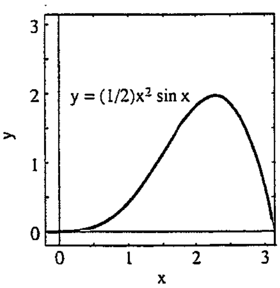

Fig. 9.3.4 The region of Problem 64

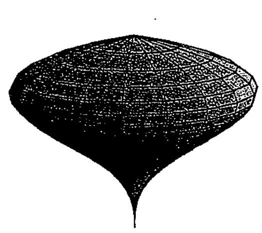

Fig. 9.3.5 The top of Problem 65

and

$$1 + \cot^2 x = \csc^2 x, \quad D_x \cot x = -\csc^2 x, \quad D_x \csc x = -\csc x \cot x.$$

The method of case 1 succeeds with the integral

$$\int \tan^n x \, dx$$

only when n is an odd positive integer, but there is another approach that works equally well whether n is even or odd. We split off the factor $\tan^2 x$ and replace it with $\sec^2 x - 1$:

$$\begin{aligned}\int \tan^n x \, dx &= \int (\tan^{n-2} x)(\sec^2 x - 1) \, dx \\ &= \int \tan^{n-2} x \sec^2 x \, dx - \int \tan^{n-2} x \, dx.\end{aligned}$$

We integrate what we can and find that

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx. \tag{12}$$

Equation (12) is another example of a reduction formula. Its use effectively reduces the original exponent from n to $n - 2$. If we apply Eq. (12) repeatedly, we eventually reduce the integral to either

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx = \tan x - x + C$$

or

$$\int \tan x \, dx = \ln |\sec x| + C.$$

EXAMPLE 9 Two applications of Eq. (12) give

$$\begin{aligned}\int \tan^6 x \, dx &= \tfrac{1}{5} \tan^5 x - \int \tan^4 x \, dx \\ &= \tfrac{1}{5} \tan^5 x - \left(\tfrac{1}{3} \tan^3 x - \int \tan^2 x \, dx \right) \\ &= \tfrac{1}{5} \tan^5 x - \tfrac{1}{3} \tan^3 x + \tan x - x + C.\end{aligned}$$

Finally, in the case of an integral involving an unusual mixture of trigonometric functions—tangents and cosecants, for example—expressing the integrand entirely in terms of sines and cosines may yield an expression that's easy to integrate.

9.4 PROBLEMS

Evaluate the integrals in Problems 1 through 44.

1. $\int \sin^2 2x \, dx$

2. $\int \cos^2 5x \, dx$

3. $\int \sec^2 \frac{x}{2} \, dx$

4. $\int \tan^2 \frac{x}{2} \, dx$

5. $\int \tan 3x \, dx$

6. $\int \cot 4x \, dx$

7. $\int \sec 3x \, dx$

8. $\int \csc 2x \, dx$

[redacted]

9. $\displaystyle\int \frac{1}{\csc^2 x}\,dx$

10. $\displaystyle\int \sin^2 x \cot^2 x\,dx$

11. $\displaystyle\int \sin^3 x\,dx$

12. $\displaystyle\int \sin^4 x\,dx$

13. $\displaystyle\int \sin^2\theta \cos^3\theta\,d\theta$

14. $\displaystyle\int \sin^3 t \cos^3 t\,dt$

15. $\displaystyle\int \cos^5 x\,dx$

16. $\displaystyle\int \frac{\sin t}{\cos^3 t}\,dt$

17. $\displaystyle\int \frac{\sin^3 x}{\sqrt{\cos x}}\,dx$

18. $\displaystyle\int \sin^3\phi \cos^4\phi\,d\phi$

19. $\displaystyle\int \sin^5 2z \cos^2 2z\,dz$

20. $\displaystyle\int \sin^{3/2} x \cos^3 x\,dx$

21. $\displaystyle\int \frac{\sin^3 4x}{\cos^2 4x}\,dx$

22. $\displaystyle\int \cos^6 4\theta\,d\theta$

23. $\displaystyle\int \sec^4 t\,dt$

24. $\displaystyle\int \tan^3 x\,dx$

25. $\displaystyle\int \cot^3 2x\,dx$

26. $\displaystyle\int \tan\theta \sec^4\theta\,d\theta$

27. $\displaystyle\int \tan^5 2x \sec^2 2x\,dx$

28. $\displaystyle\int \cot^3 x \csc^2 x\,dx$

29. $\displaystyle\int \csc^6 2t\,dt$

30. $\displaystyle\int \frac{\sec^4 t}{\tan^2 t}\,dt$

31. $\displaystyle\int \frac{\tan^3\theta}{\sec^4\theta}\,d\theta$

32. $\displaystyle\int \frac{\cot^3 x}{\csc^2 x}\,dx$

33. $\displaystyle\int \frac{\tan^3 t}{\sqrt{\sec t}}\,dt$

34. $\displaystyle\int \frac{1}{\cos^4 2x}\,dx$

35. $\displaystyle\int \frac{\cot\theta}{\csc^3\theta}\,d\theta$

36. $\displaystyle\int \sin^2 3\alpha \cos^2 3\alpha\,d\alpha$

37. $\displaystyle\int \cos^3 5t\,dt$

38. $\displaystyle\int \tan^4 x\,dx$

39. $\displaystyle\int \cot^4 3t\,dt$

40. $\displaystyle\int \tan^2 2t \sec^4 2t\,dt$

41. $\displaystyle\int \sin^5 2t \cos^{3/2} 2t\,dt$

42. $\displaystyle\int \cot^3\xi \csc^{3/2}\xi\,d\xi$

43. $\displaystyle\int \frac{\tan x + \sin x}{\sec x}\,dx$

44. $\displaystyle\int \frac{\cot x + \csc x}{\sin x}\,dx$

In Problems 45 through 48, find the area of the region bounded by the two given curves.

45. The x-axis and the curve $y = \sin^3 x$, from $x = 0$ to $x = \pi$

46. $y = \cos^2 x$ and $y = \sin^2 x$, from $x = -\pi/4$ to $x = \pi/4$

47. $y = \sin x \cos x$ and $y = \sin^2 x$, from $x = \pi/4$ to $x = \pi$

48. $y = \cos^3 x$ and $y = \sin^3 x$, from $x = \pi/4$ to $x = 5\pi/4$

In Problems 49 and 50, first graph the integrand function and guess the value of the integral. Then verify your guess by actually evaluating the integral.

49. $\displaystyle\int_0^{2\pi} \sin^3 x \cos^2 x\,dx$

50. $\displaystyle\int_0^{\pi} \sin^5 2x\,dx$

In Problems 51 through 54, find the volume of the solid generated by revolving the given region R around the x-axis.

51. R is bounded by the x-axis and the curve $y = \sin^2 x$, $0 \leqq x \leqq \pi$.

52. R is the region of Problem 46.

53. R is bounded by $y = 2$ and $y = \sec x$ for $-\pi/3 \leqq x \leqq \pi/3$.

54. R is bounded by $y = 4\cos x$ and $y = \sec x$ for $-\pi/3 \leqq x \leqq \pi/3$.

55. Let R denote the region that lies between the curves $y = \tan^2 x$ and $y = \sec^2 x$ for $0 \leqq x \leqq \pi/4$. Find: (a) the area of R; (b) the volume of the solid obtained by revolving R around the x-axis.

56. Find the length of the graph of $y = \ln(\cos x)$ from $x = 0$ to $x = \pi/4$.

57. Find

$$\int \tan x \sec^4 x\,dx$$

in two different ways. Then show that your two results are equivalent.

58. Find

$$\int \cot^3 x\,dx$$

in two different ways. Then show that your two results are equivalent.

Problems 59 through 62 are applications of the trigonometric identities

$$\sin A \sin B = \tfrac{1}{2}[\cos(A - B) - \cos(A + B)],$$

$$\sin A \cos B = \tfrac{1}{2}[\sin(A - B) + \sin(A + B)],$$

$$\cos A \cos B = \tfrac{1}{2}[\cos(A - B) + \cos(A + B)].$$

59. Find $\displaystyle\int \sin 3x \cos 5x\,dx$.

60. Find $\displaystyle\int \sin 2x \sin 4x\,dx$.

61. Find $\displaystyle\int \cos x \cos 4x\,dx$.

62. Suppose that m and n are positive integers with $m \neq n$. Show that (a) $\int_0^{2\pi} \sin mx \sin nx\,dx = 0$; (b) $\int_0^{2\pi} \cos mx \sin nx\,dx = 0$; (c) $\int_0^{2\pi} \cos mx \cos nx\,dx = 0$.

63. Substitute $\sec x \csc x = (\sec^2 x)/(\tan x)$ to derive the formula

$$\int \sec x \csc x\,dx = \ln|\tan x| + C.$$

64. Show that

$$\csc x = \frac{1}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)},$$

then apply the result of Problem 63 to derive the formula

$$\int \csc x \, dx = \ln\left|\tan \frac{x}{2}\right| + C.$$

65. Substitute $x = \frac{1}{2}\pi - u$ into the integral formula of Problem 64 to show that

$$\int \sec x \, dx = \ln\left|\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right| + C.$$

66. Use appropriate trigonometric identities to deduce from the result of Problem 65 that

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C.$$

67. Show first that the reduction formula in Eq. (12) gives

$$\int \tan^4 x \, dx = \tfrac{1}{3}\tan^3 x - \tan x + x.$$

Then compare this result with the alleged antiderivative

$$\int \tan^4 x \, dx = \tfrac{1}{12}(\sec^3 x)(9x \cos x + 3x \cos 3x - 4 \sin 3x)$$

given by some versions of *Mathematica.*

68. Compare the result given in Example 9 with the integral

$$\int \tan^6 x \, dx$$

as given by your favorite computer algebra system.

9.5 RATIONAL FUNCTIONS AND PARTIAL FRACTIONS

We now discuss methods with which every rational function can be integrated in terms of elementary functions. Recall that a rational function $R(x)$ is a function that can be expressed as the quotient of two polynomials. That is,

$$R(x) = \frac{P(x)}{Q(x)}, \tag{1}$$

where $P(x)$ and $Q(x)$ are polynomials. The **method of partial fractions** is an *algebraic* technique that decomposes $R(x)$ into a sum of terms:

$$R(t) = \frac{P(x)}{Q(x)} = p(x) + F_1(x) + F_2(x) + \cdots + F_k(x), \tag{2}$$

where $p(x)$ is a polynomial and each expression $F_i(x)$ is a fraction that can be integrated with little difficulty.

EXAMPLE 1 We can verify (by finding a common denominator on the right) that

$$\frac{x^3 - 1}{x^3 + x} = 1 - \frac{1}{x} + \frac{x - 1}{x^2 + 1}. \tag{3}$$

It follows that

$$\int \frac{x^3 - 1}{x^3 + x} \, dx = \int \left(1 - \frac{1}{x} + \frac{x}{x^2 + 1} - \frac{1}{x^2 + 1}\right) dx$$
$$= x - \ln|x| + \tfrac{1}{2}\ln(x^2 + 1) - \tan^{-1} x + C.$$

The key to this simple integration lies in finding the decomposition given in Eq. (3). The existence of such a decomposition and the technique of finding it are what the method of partial fractions is about. See Fig. 9.5.1. ■

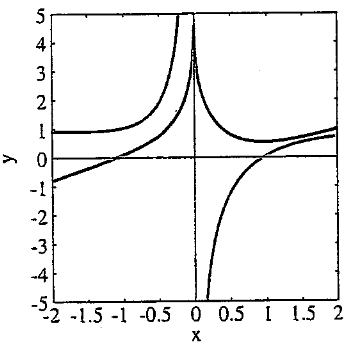

Fig. 9.5.1 Graphs of the function $f(x) = (x^3 - 1)/(x^3 + x)$ of Example 1 and its indefinite integral with $C = 0$. Which is which?

According to a theorem proved in advanced algebra, every rational function can be written in the form in Eq. (2) with each $F_i(x)$ being a fraction either of the form

$$\frac{A}{(ax + b)^n} \tag{4}$$

[redacted]