Real Analysis

Homework 4

due October 1, 2008

1. (From Lieb and Loss.) Write out a simple sufficient condition on f_n such that

$$\sum_{n=0}^{\infty} \int_{\Omega} f_n(x)\, \mathrm{d}\mu(x) = \int_{\Omega} \sum_{n=0}^{\infty} f_n(x)\, \mathrm{d}\mu(x)\,.$$

2. Let $(\Omega_i, \Sigma_i, \mu_i)$ for $i = 1, 2, 3$ be σ-finite measure spaces. Show that the product measure is associative, i.e.
$$\mu_1 \times (\mu_2 \times \mu_3) = (\mu_1 \times \mu_2) \times \mu_3\,.$$

   Note: A sketch of the proof is given in Lieb and Loss, Corollary 1.11. Write out a detailed argument.

3. (From Lieb and Loss.) Give a "counterexample" to Tonelli's theorem in the absence of σ-finiteness.

   Hint: Take the Lebesgue measure on $[0, 1]$ as one measure space and the counting measure on $[0, 1]$ as the other.

4. Give a "counterexample" to Fubini's theorem in the absence of summability.