Real Analysis

Homework 2

due September 15, 2008

1. (From Lieb and Loss.) Let (Ω, Σ, μ) be a measure space. It is natural to define $\mu(A) = 0$ if A is the subset of a nullset, even though A may not be contained in the σ-algebra.

   Show that if we extend Σ by adding or subtracting such sets of measure zero to the sets in Σ, the resulting collection of sets is again a σ-algebra, and the extended measure is again a measure on the larger σ-algebra.

2. Complete the proof of Theorem 1.4 in Lieb and Loss by writing out the proof for the σ-finite case, filling in the two missing arguments marked "Why?" in the text.

3. (From Folland.) Let μ denote the Lebesgue measure on the real line. If $E \in \mathcal{B}_{\mathbb{R}}$ and $\mu(E) < \infty$, then for every $\varepsilon > 0$ there exists a set A which is a finite union of open intervals such that
$$\mu((E \setminus A) \cup (A \setminus E)) < \varepsilon \,.$$