Real Analysis

Homework 11

due November 26, 2008

1. Compute the Fourier transform on $\mathbb{R}$ of $\chi_{[-1,1]}(x)$.

2. (From Folland.) Let $f(x) = x$ on $[-\pi, \pi)$, interpreted as a function on $\mathbb{T}$.

   (a) Compute the Fourier series of f.

   (b) Show that

   $$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}\,.$$

   Hint: Parseval identity.

3. Prove the *Riemann–Lebesgue lemma* for Fourier series, i.e. for $f \in L^1(\mathbb{T}^n)$, $\hat{f}_k \to 0$ as $|k| \to \infty$.

4. (From Folland.) Let

   $$D_m(x) = \frac{1}{2\pi} \sum_{|k| \le m} \mathrm{e}^{\mathrm{i}kx}$$

   be the mth Dirichlet kernel. Show that

   $$\|D_m\|_{L^1(\mathbb{T})} \to \infty \qquad \text{as} \qquad m \to \infty\,.$$