Partial Differential Equations

Homework 4

due October 8, 2008

1. Let U be open and bounded with a C^1 boundary. For every $v \in C^2(\bar{U})$, set

$$J[v] = \int_U \left(\tfrac{1}{2}\,|Dv|^2 - f\,v\right) dx - \int_{\partial U} g\,v\,dS\,.$$

Assume thoughout that $u \in C^2(\bar{U})$. Prove that the following two statements are equivalent.

(i) u solves the so-called *Neumann problem*

$$\begin{aligned} -\Delta u &= f \qquad \text{in } U\,, \\ \nu \cdot Du &= g \qquad \text{on } \partial U\,. \end{aligned}$$

(ii) u minimizes J, i.e.

$$J[u] \le J[w]$$

for every $w \in C^2(\bar{U})$.

2. Evans, p. 87 problem 10

3. Evans, p. 87 problem 11

4. Evans, p. 87 problem 12