

Partial Differential Equations

Homework 1

due September 10, 2008

1. Evans, p. 85 problem 1
2. Evans, p. 85 problem 2
3. In the proof of the theorem on the solution of Poisson's equation we have used that

$$D \int_{\mathbb{R}^n} \Phi(y) f(x - y) dy = \int_{\mathbb{R}^n} \Phi(y) Df(x - y) dy.$$

State why and precisely under which assumptions this manipulation is permitted.

4. Consider a function of one complex variable $w: \mathbb{C} \rightarrow \mathbb{C}$ on an open connected subset of the complex plane, and write $w = w(z)$ with $w = u + iv$ and $z = x + iy$. The function w is called (*complex*) *differentiable* or *holomorphic* if u and v have continuous first partial derivatives with respect to x and y that satisfy the so-called *Cauchy–Riemann equations*

$$\begin{aligned}u_x &= v_y \\u_y &= -v_x\end{aligned}$$

It is known that holomorphic functions are infinitely differentiable.

Show that the real and imaginary parts of a holomorphic function are harmonic.