General Mathematics and Computational Science I

Midterm II

November 13, 2007

1. How many distinct subsets does a finite set with n elements have? (The empty set and the set itself are should be included in the count.) (7)

2. Three dice are tossed.

   (a) What is the probability of obtaining at least one 6?

   (b) What is the probability of obtaining at most one 6?

   (5+5)

3. Use the method of generating functions to find a closed form expression for the members of the sequence
$$a_n = \tfrac{5}{2}\, a_{n-1} - a_{n-2}$$
where $a_0 = 2$ and $a_1 = \frac{5}{2}$. (10)

4. Show that, among all right triangles with hypotenuse of length 1, the isosceles triangle has the largest area. (8)

5. Show that, for arbitrary numbers $a_1, \ldots, a_n$,
$$\sum_{k=1}^{n} a_k \leq n^{\frac{1}{2}} \left(\sum_{k=1}^{n} a_k^2 \right)^{\frac{1}{2}} .$$
(7)

6. In class, we computed the probability P that among a group of k people at least two are born on the same day of the year. In the derivation, we assumed that that each day has equal probability of being the birthday of someone. In reality, however, there are weekly and seasonal variations of the distribution of births. Would, under more realistic assumptions on the distribution of births, the quantity P increase or decrease?

Note: No credits without explanation. Full credit for a convincing qualitative argument. Up to five extra credits for a complete proof. (8+5)