Partial Differential Equations

Homework 6

due March 21, 2007

1. Evans, p. 88 problem 14

2. Prove a maximum principle for the following semilinear PDE, called Burger's equation,

$$u_t + u\,u_x = u_{xx}\,,$$
$$u(x,0) = g(x)$$

   where $u = u(x,t)$ and $(x,t) \in \mathbb{R} \times [0,\infty)$.

3. Find the Fourier transforms for the following functions on $\mathbb{R}$:

   (a) $f(x) = e^{-t|x|}$,

   (b) $f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$.

4. The so-called *Helmholtz equation* on $\mathbb{R}$,

$$u - u_{xx} = f\,,$$

   is similar to the Poisson equation: Its solution can be written in the form

$$u(x) = \int_{\mathbb{R}} \Psi(x-y)\, f(y)\, dy\,.$$

   Use the Fourier transform and the result from (3a) to find Ψ.