Partial Differential Equations

Homework 5

due March 14, 2007

1. Finish the proof of the mean value formula for the heat equation by showing that

$$\iint_{E(0,0;1)} \frac{|y|^2}{s^2}\, dy\, ds = 4\,,$$

   where

$$E(x,t;r) = \{(y,s)\colon \Phi(x-y,t-s) \geq r^{-n}\}$$

   denotes the heat ball "centered" at (x,t).

   Hint: Use polar coordinates in space, and an appropriate change of variables in time. The remaining one-dimensional integral is Mathematica-integrable. You can also use that

$$\int_0^\infty t^{\lambda+1}\, e^{-\lambda t}\, dt = \frac{\Gamma(\lambda+2)}{\lambda^{2+\lambda}}\,,$$
$$\Gamma(x+1) = x\,\Gamma(x)\,,$$
$$\alpha(n) = \frac{\pi^{n/2}}{\Gamma(\frac{n}{2}+1)}\,.$$

2. Evans, p. 87 problem 12

3. Evans, p. 87 problem 13