

General Mathematics and Computational Science II

Exercise 20

May 7, 2007

Let G be a finite abelian group of order n (i.e. G has n elements), written additively. Then $\chi: G \rightarrow \mathbb{C} \setminus \{0\}$ is a *character* of G if

$$\chi(a + b) = \chi(a) \chi(b)$$

for all $a, b \in G$.

1. Show that

$$\chi(-a) = \chi(a)^{-1} = \overline{\chi(a)}$$

for all $a \in G$.

Hint: Recall that the values taken by a character are n th roots of unity.

2. The set of characters of G is called the *dual group*, denoted \hat{G} . Show that it is indeed an abelian group when the group operation is given by the product

$$(\chi\psi)(a) = \chi(a) \psi(a).$$