General Mathematics and Computational Science II

Exercise 17

April 24, 2007

1. Show that for every sequence a_n of nonnegative real numbers there exists some $c > 0$ such that
$$\sum_{n=1}^{\infty} a_n^2 \leq c \sum_{n=1}^{\infty} a_n \,.$$
Can you give an estimate of the dependence of c on a_n?

2. Let
$$v(x) = \sum_{k=-\infty}^{\infty} \hat{v}_k \, \mathrm{e}^{\mathrm{i}kx} \,.$$
Show that
$$\hat{v}_k = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{e}^{-\mathrm{i}kx} \, v(x) \, \mathrm{d}x \,.$$