

General Mathematics and Computational Science II

Exercise 4

February 13, 2007

1. Show that

$$\sum_{k=0}^n \binom{n}{k} k x^k y^{n-k} = n x (x + y)^{n-1}$$

and

$$\sum_{k=0}^n \binom{n}{k} k^2 x^k y^{n-k} = n x (x + y)^{n-1} + n (n - 1) x^2 (x + y)^{n-2}.$$

Hint: Differentiate the binomial theorem.

2. Consider an ensemble of Kac rings with N sites where each site is colored black independently with probability ν . Show that

$$\langle \Delta \rangle = (2\nu - 1) N$$

and

$$\text{Var}[\Delta] = 4\nu(1 - \nu) N.$$

Hint: Use the result of the previous question.

Remark: This is in fact a direct consequence of a well known result on the variance of the binomial distribution which has a rather elegant proof in terms of *moment generating functions*. These are usually introduced in a first course on probability theory. You will likely encounter them this semester in Engineering and Science Mathematics 2A/B.