General Mathematics and Computational Science II

Exercise 1

February 1, 2007

1. Newton's second law of mechanics for a particle of mass m situated at position $x(t)$ moving with velocity $v(t)$ and subject to a force $F(x(t))$ can be written

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v \,,$$

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = F(x(t)) \,.$$

   Use the chain rule of calculus to show that the particle satisfies the same equation with t replaced by the reversed time $r = -t$ and v replaced by $-v$.

2. In the Kac ring model, N sites are placed around a circle. The sites are populated with B black balls and $W = N - B$ white balls at random. Moreover, n markers are placed on the edges between the sites at random; the number of blace balls just before a marked edge is denoted b, the number of white balls just before a marked edge is w.

   Let μ denote the probability that an edge has a marker on it. Explain why

$$\mu = \frac{n}{N} = \frac{b}{B} = \frac{w}{W} \,.$$