Math, Mind, and Music

Quiz 1

February 25, 2005

1. Match the descriptions of spectra

   (a) Fundamental and all harmonics have approximately equal strength

   (b) Fundamental is strong, higher harmonics present but decreasing in strength

   (c) Only harmonics 1,2,3 and 10,11,12 are present

   (d) Only harmonics 1,2,3 and 6,7,8 are present

   (e) All frequencies above the fundamental (not only harmonics) are strong

   with the following corresponding subjective impressions of the sound:

   (1) Percussive/noisy

   (2) Dissonant

   (3) Full

   (4) Sharp

   (5) Two clearly distinct tones

   (5)

2. Answer either (a) or (b), not both.

   (a) Recall that the open end of a pipe is always a region of maximal wave amplitude, while the closed end of a pipe is always a region of zero motion. Does a pipe which is closed at both ends produce a full set of harmonics? Explain!

   (b) Why do the sounds of two instruments always reinforce, and never cancel out?

   (5)

3. Consider the following question:

   Is the human auditory system's ability to discriminate frequencies different when presented with *purely sinusoidal tones* as opposed to *natural tones* having a full set of harmonics?

(a) State and explain your expectation with regard to the *just noticeable difference*—the smallest perceivable difference in frequency between two consecutive single tones of equal loudness.

(b) State and explain your expectation with regard to the *limit of frequency discrimination*—the smallest difference in frequency at which two tones played simultaneously are perceived as separate.

(c) Describe an experiment to test your expectation for either (a) or (b).

(3+3+4)

Solutions

1. a4, b3, c5, d2, e1

2. (a) Yes. The fundamental mode clearly has zero displacement at both ends, and a single region of maximal amplitude at the center. If you divide the pipe in two equal halfs, then each of the halfs supports a pattern of vibration that looks like the fundamental of the entire pipe. The same goes for a division in three, four, etc. As this corresponds to a doubling, tripling, etc. of the frequency, all harmonics can be realized. (In fact, this is exactly the same situation as for a string.)

   (b) Quote from the *FAQ in music acoustics*, the University of New South Wales, visited March 3, 2005:

   "The answer to this involves several different effects that complicate the sound of musical instruments. To hear the effect of destructive interference, you have first to eliminate each of these effects, and it is rare that they are all eliminated together, which is why you don't normally hear destructive interference in practice. Neverthe less, when two instruments are nearly but not exactly in tune, you do hear the phenomenon of beats (listen to the sound files of beats). This is an example of constructive and destructive interference: the slight difference in frequency causes the phase relationship to change slowly. When the two waves are in phase it sounds loud but, when they are out of phase it is soft.

   Beats between real musical instruments do have variation in loudness, but the loudness doesn't usually go to zero. There are several reasons for this:

   - First, real musical instruments rarely have exactly equal amplitude, so even when they are exactly out of phase, their waves don't cancel. (And they often have vibrato.) Even if they were of equal amplitude, it is unlikely that you would be equally distant from both. Further, this effect is enhanced by the logarithmic response of your ears (see *what is a decibel?* for details). If two waves cancel to 99%, the resulting sound is 40 dB softer than one instrument alone. Listeners often judge that 40dB softer sounds about 1/16 as loud. So even if you achieve cancellation with this precision (hard to do), the effect is not as spectacular as you might expect.

- Second, real musical instruments don't play sine waves: they have several harmonics. Even when the conditions are such that two harmonics cancel, other harmonics may not.
- Third: unless you are in an anechoic chamber, the points of cancellation are not where you calculate them to be, because of multiple reflections off walls.
- Fourth: you have two ears. Even if one ear is at the point of cancellation, the other is not.
- Fifth: your ears are most sensitive in the range 1–4 kHz, which means wavelengths of $\sim$300 mm to $\sim$100 mm. Nearly all instruments radiate fairly strongly in this range, so you must get cancellation in this frequency range to get cancellation. However, because of these short wavelengths the region of cancellation (if there is one, in spite of the complications mentioned above) is very small, and the chance of having even one ear at the point is small.
- Sixth: instruments and peoples ears move on a scale of at least 10s of mm (often more) during performance.

Despite all of the above, it is possible to set up conditions under which you can experience the interference effects. Simply set up a sine wave source (eg. an electronic tuner) in a room. There will be reflections off walls and other objects that cause the amplitude to vary strongly from place to place. Cover one ear, put the other near a wall, and move your head towards and away from it. If you were to drive two identical speakers with the same signal but reversed in phase, and if you did it in an anechoic chamber, then you should get cancellation on the plane of symmetry between the speakers. If you put one ear on this plane, and neglecting the reflections from your body, you'd expect to get pretty good cancellation. (I've never tried this experiment in an anechoic chamber, but I've certainly noticed the effect of reflections from walls.)

In Berio's *Sequenza VII*, a sine wave is played throughout on the note B4. It creates an eery ambiance: one doesn't know where it is coming from and it seems to get louder and softer, for several of the reasons discussed above, including the motion of performer and audience."

`http://www.phys.unsw.edu.au/~jw/musFAQ.html#interference`

3. (a) The JNR curve grows much less than by a factor of two per octave over a large portion of the audible range (see powerpoint slides). Therefore, even in cases where the difference of the fundamentals is below the JNR, the difference of one of the higher harmonics may exceed it. Moreover, the JNR actually increases at very low frequencies. Thus, comparing natural tones should typically be easier than comparing pure sinusoidal tones. Note, however, that a very strong fundamental may mask this effect.

   (b) The limit for frequency discrimination is roughly 30 times larger than the JND, so the same argument applies.

(c) We'll describe an experiment for the JNR; an experiment for the corresponding limit of frequency discrimination would be similar.

First, for a given test subject and fundamental frequency, determine the JNR for pure sinusoidal tones by a separate experiment, or use a reasonable value from the literature. Next, choose a difference of the fundamental frequencies which is just below the JNR. We can then generate a sequence of pairs of tones in which an increasing portion of the energy is contained in higher harmonics. There are obviously many ways to do this; an elegant way is to have the energy per mode, i.e. the square of the amplitude of a particular mode, Poisson-distributed. If our conjecture from (a) holds, there should be a point at which the subject is able to reliably compare the pitch of the two tones in a pair. Randomization of the order in which the samples are presented will help reduce adaptation effects and interference of expectation.