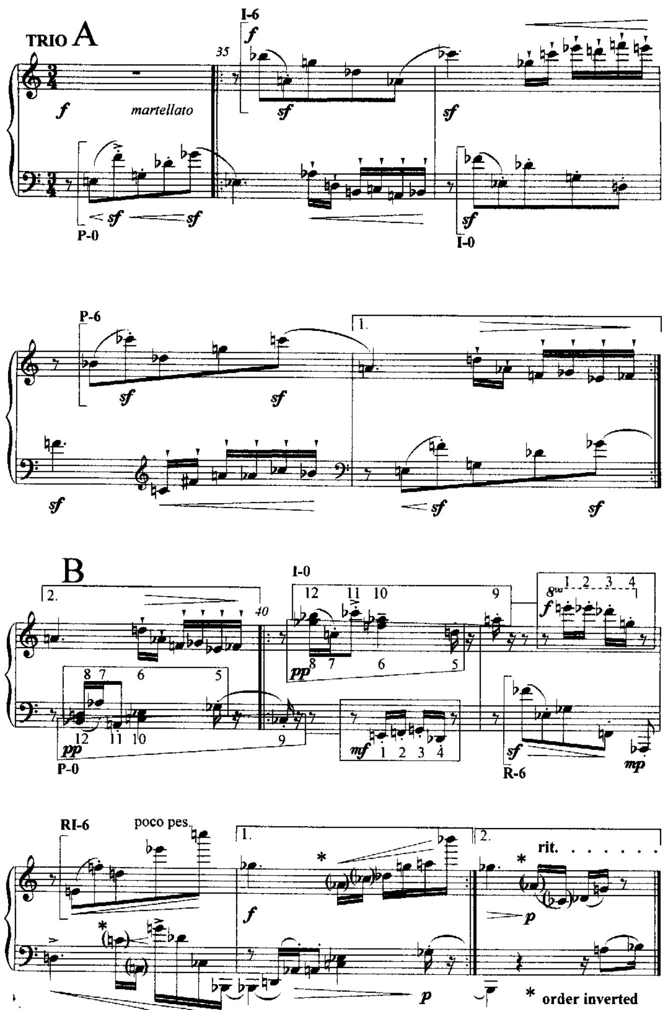


The opening of the Trio from the Minuet and Trio of Schoenberg's *Piano suite*, Op. 25, showing the disposition of the six forms of the basic tone row used in its composition.

CHAPTER 8

Composing with numbers: sets, rows and magic squares

Jonathan Cross

Throughout the twentieth century mathematical ideas emerged as basic tools for the composer. Here we consider a range of these, from the twelve-tone row of Arnold Schoenberg and the magic squares of Peter Maxwell Davies to the use of set theory and geodesic surfaces by Iannis Xenakis.

Accusations of lack of artistry, lack of creative imagination, and even lack of musicality have been hurled by critics and music-lovers alike at very many twentieth-century composers, and not least at the Viennese composer, Arnold Schoenberg. His 'discovery', as he put it, in the early 1920s, of his 'method of composing with twelve tones', was seen by those at a distance from his work as being a kind of compositional equivalent of those 'painting by numbers' kits that can be bought in children's toy shops. In Schoenberg's composing kit was to be found the composer's equivalent of paint and brushes—namely, the twelve notes of the chromatic scale, arranged in any order of the composer's choosing, so long as each note appeared only once (a 'tone row'). The overprinted canvas—for Schoenberg—was often a ready-made form from the musical past: a movement from a Baroque suite, a waltz, or even a sonata form movement. Onto this canvas the tone row was laid, according to very straightforward mathematical operations of translation (or, in musical terms, transposition) and different kinds of mirroring (inversion and retrogradation).

Put in this way, it is hardly surprising that Schoenberg has recurrently been misrepresented as the bogeyman of twentieth-century music. Nineteenth-century Romantic thought had led us to believe that the composer was someone special, almost a god, set apart from the rest of ordinary society. He was someone in touch with the muses who waited for inspiration to strike before pouring out his soul, by means of some mystical process, in order to produce works of art to be revered by the masses almost as if they were holy relics—a surprising attitude, one might think, for an age which paradoxically saw the rapid development of logical scientific knowledge and method. Schoenberg *seemed* to be suggesting quite the opposite. Twelve-note music had nothing to do with inspiration, or even with musicality, but was seen as mechanical or, worse still, mathematical.

However, this is a misguided view. As will be seen below, many composers of the twentieth century found numbers and various mathematical models a useful source of compositional material or of processing material. In the hands of some, the results certainly are mundane and mechanical. But mundane and mechanical music is possible under any system—not least, tonality. It is the creative use to which such number systems are put that makes for a 'successful' piece of music, not the fact that numbers in themselves have been compositionally deployed.

Why, then, did Schoenberg *feel* it necessary to invent the twelve-note method? The answer to this question should tell us much, not only about Schoenberg's peculiar historical predicament, but also about why so many composers in recent decades have attempted to frame their music within the context of mathematics.

Arnold Schoenberg

Schoenberg presided over the break-up of tonality, the system that had governed the composition of music for 300 years. When, in 1907, he finally abandoned a key-signature in the finale of his *2nd string quartet*, it was not a wilful attempt to destroy the past; rather, it was an inevitable and necessary step. Tonality had reached the end of its useful life; it could no longer contain the extreme levels of chromaticism and dissonance that had developed in music. The dissonance had to be emancipated.

But with the abandonment of tonality, Schoenberg was confronted with the problem that nearly all composers of the twentieth century had to face. Where was he now to begin? There was no obvious context, no common practice within which to start writing. With every piece he had to begin afresh, had to create his own rules and modes of operation, his own structures. At first he was able to write only very short or fragmentary pieces, or was compelled to rely on texts to structure the music. But eventually he moved to a position where he began to use contrapuntal techniques to provide a more *logical* structure, and eventually this became codified in the twelve-note system. His aim in adopting the 'method' was to provide *comprehensibility* (out of the 'chaos' of free atonality), its main advantage, he claimed, being its unifying effect: 'In music there is no form without logic, there is no logic without unity'. The rigour, the mathematical logic, of the twelve-note system was, in some senses, a substitute for the logical rules of the tonal system.

However, and this is perhaps the most important thing, Schoenberg did not see the method as a general panacea for the ills of twentieth-century music. Far from it: 'The introduction of my method of composing with twelve tones does *not* facilitate composing'. The method merely provided a logical context within which composition could take place. As he wrote, 'One has to follow the basic set; but, nevertheless, one composes as before', a view echoed almost exactly by his pupil Webern, 'For the rest, one composes as before, but on the basis of the row'.

Composers of the twentieth century found many ways around this central problem. Some adopted and adapted Schoenberg's method; others, as we shall see, drew on mathematical sources such as set theory, game theory, magic squares, Fibonacci numbers, and so on, to provide them with material or methods of working. Neither the method, nor mathematics, nor any other system, has made the actual act of composition any easier, nor (necessarily) any more mechanical.

Let us consider an example of Schoenberg's twelve-note practice: the Trio from the Minuet and Trio of the *Piano suite*, Op. 25 (1921–3). Figure 1 shows all the material for the Trio. The form of each row is indicated by a letter: P = prime (or original), I = inversion, R = retrograde (the prime form backwards) and RI = retrograde inversion (inversion backwards). Of the 48 possible forms of his twelve-note row, Schönberg uses just six in the Trio: the row itself (*P-0*), the row transposed up a tritone (*P-6*), this transposition backwards (*R-6*), the inversion of the row (*I-0*) and its tritonal transposition and retrograde (*I-6* and *RI-6*).

Already Schoenberg is having fun with the peculiar properties of this row, with certain patterns that remain constant across the geometric transformations. For instance, the row spans a tritone from E (note 1) to B♭ (note 12), so that by employing transpositions only of a tritone, each form of the row will begin with either an E or a B♭. This interval will then become explicitly represented in the music when, as happens in the

Figure 1. The six forms of the row used in the Trio.

Trio, the six forms of the row are strung together. Another invariant across the transformations is a further tritonal pair G–D♭ (symmetrically placed within the E–B♭ pair), and a feature is made of this in the music.

So how does this manifest itself compositionally? The extract at the beginning of the chapter shows us. What we see and hear is a beautifully formed piece of geometry realised in music. The Trio is canonic throughout and the structure of the row and its transformations articulates these canons clearly. In the first half we have a canon at the tritone in inversion at a bar's distance: *P-0* imitated by *I-6*, and *I-0* imitated by *P-6*. The second half splits the row into groups of four, still an inversional canon but now at the octave (*P-0* imitated by *I-0*). Finally, we return to two-part counterpoint where *R-6* is imitated by *RI-6*.

Thus, though Schoenberg has followed the basic set throughout, nevertheless, in terms of transpositions and deployment, and in terms of rhythms, registers, dynamics and form, Schoenberg has composed freely. The row provides intervallic material; it does not do the composer's work for him.

Interestingly, Schoenberg was not the first to invent a twelve-note system—such ideas were evidently 'in the air' in Vienna in the earlier years of the century. Josef Matthias Hauer, a Viennese contemporary of Schoenberg, had already devised a different system of composing with all twelve notes before Schoenberg. Hauer's ideas were based on what he described as cosmic laws, and (notably) he proposed that music—specifically, atonal music—represented a supreme kind of mathematics.

Alban Berg

Schoenberg's pupils quickly followed their teacher's example by adopting the twelve-note method. The first substantial work of Alban Berg's to use the method (although not in every movement) is the *Lyric suite* for string quartet of 1926. The outer sections of the 3rd movement, the 'allegro misterioso', employ the method: indeed, its structure is dependent on a simple mirroring device where two-thirds of the first 69 bars of the movement are mirrored *exactly* in the last 46 bars and frame a central, more freely atonal section of 23 bars (see Figure 2).

These numbers are highly significant because there is another sense in which Berg was 'composing with numbers' in the *Lyric suite*, and this has to do with its proportional relations: both the durational lengths and the tempi of movements. There are various numerological clues in the score, but the extent to which number symbolism, as well as other kinds of cryptograms and enigmatic quotations, govern the structure of the work, was first fully revealed by George Perle in 1977. Berg, it seems, was obsessed with the number 23. It evidently had some great

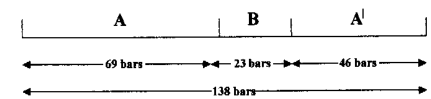


Figure 2. The proportions of the third movement of Berg's *Lyric suite* are governed by Berg's 'fateful' number 23.

personal significance for him—he referred to it as his 'fateful' number. If we look again at the 'allegro misterioso', we can see that its proportions, in terms of numbers of bars, are governed by multiples of 23. This is no fluke. Movements 1 and 4 are both 69 bars long (3 × 23); movement 5 is 460 bars long (20 × 23); and movement 6 is 46 bars long (2 × 23). Furthermore, the metronome markings are also multiples of 23: movement 4, ♩ = 69; movement 6, ♩ = 69 and ♩ = 46.

As for the length and tempi of the other movements, they are all multiples of 10: metronome marks of 100 or 150 and a second movement that is 150 bars long. Note that the length of movement 5 (460 bars) is a multiple of both 23 and 10. What is the significance of this 10? Some detective work by Perle, including a reading of Berg's letters and the discovery of a miniature score meticulously and colourfully annotated by Berg, revealed that 10 was the fateful number of Mrs. Hanna Fuchs-Robettin with whom, it transpires, Berg had become passionately involved. The score is secretly dedicated to her in Berg's own hand:

> It has also, my Hanna, allowed me other freedoms! For example, that of secretly inserting our initials, HF and AB, into the music, and of relating every movement and every section of every movement to our numbers, 10 and 23. I have written these, and much that has other meanings, into the score for you ... May it be a small monument to a great love.

Thus the intertwining of 10 and 23 has not only structural implications for the composer but strongly extra-musical (extra-marital?) ones too. It remains a fascinating personal example of composing with numbers.

Anton Webern

The late works of Anton Webern, Schoenberg's other celebrated pupil, are concise statements and show a highly developed understanding of the possibilities of the twelve-note method, particularly in terms of their concentrated motivic working and their exploration of symmetrical structures. Canons abound. Yet the end results are not in any sense mechanical or abstractly mathematical but poignantly expressive. As one commentator has observed about Webern's serial string quartet: 'its "suitability for study", as a compendium of Webern's serial technique in full maturity, should not blind us to its *musical* qualities'.

The twelve-note row with which Webern composed his *Concerto*, Op. 24 (1934), is given in Figure 3a. It is a marvellous example of symmetry, even within the row itself. Each half of the row involves a

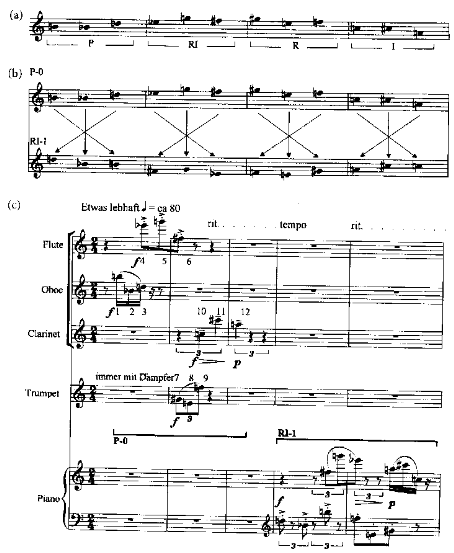


Figure 3. Webern's *Concerto*, op. 24.
(a) The basic row, showing its four three-note subsets.
(b) The *P*-0 and *RI*-1 forms of the row, showing the identical pitch-class content of each three-note set.
(c) The opening shows the use of the *P*-0 and *RI*-1 forms of the row divided into three-note groups, each containing a semitone and a major third.

mirror symmetry and the row can be further broken down into four groups of three notes, each of which contains the intervals of a semitone and a major third, and which represent, in microcosm, the four different forms of the basic row—prime or original, retrograde inversion, retrograde and inversion. Furthermore, the retrograde inversion form of the complete row in transposition (a semitone) preserves the pitch-class content of the three-note groups (see Figure 3b). Figure 3c, the opening of the first movement, shows how this is exploited in the actual music. Notice how Webern makes a rhythmic feature of the three-note groups.

Thus, numbers again provide a context within which the composer can work; they are in no sense the end result—that is, what the piece is about—which is more than can be said for the ways in which Webern was interpreted by some of the younger generation of avant-garde composers after the Second World War. The works of Webern, not Schoenberg, were viewed as the models for the future of music. Only total organisation of music in all its aspects (pitch, duration, mode of attack, dynamics, form) meant that the composer, in theory, was in complete control of the music and independent of forms and processes

from the past. Olivier Messiaen was one of the first to suggest the possibilities of total serialism in his *Modes de valeurs et d'intensités* for piano of 1949, but it was his pupil, Pierre Boulez, who took these ideas to their extreme and logical conclusion in his *Structures* for two pianos of 1952. Karlheinz Stockhausen and, on the other side of the Atlantic, Milton Babbitt were similarly extending serial principles beyond the domain of pitch.

Pierre Boulez

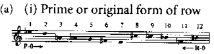
The very title of Boulez's *Structures* gives away its central premise—namely, that it is concerned with building integrated musical structures that stand on their own terms rather than being dependent on anything outside of themselves. The architectural implications of the title were intentional and exemplify a more general trend (and not just in music) towards associating art with science, mathematics and architecture. The development of the possibilities of electronics in music was just one reason for this—and the concomitant scientific exploration of the properties of sound. Varèse anticipated this in works with such titles as *Density 21.5*, *Ionisation* and *Hyperprism*. Later composers made explicit use of these ideas in works with such titles as Cage's *First construction (in metal)*, Boulez's *Polyphonie X* and Stockhausen's *Zeitmasse*. All these works represented a desire on the part of the composers to move forward, to eradicate the past and memories of earlier music; the apparent 'objectivity' of number, mathematics and the mathematically based architecture of a figure like Le Corbusier provided a means to achieve this.

The structure of Boulez's *Structures* is based entirely on the basic row from Messiaen's *Mode de valeurs*—see Figure 4a. Two number matrices were derived from this to represent all 48 forms of the row which are used once each in *Structure Ia*. Each pitch class corresponds to the same integer throughout: E♭ = 1, D = 2, A = 3, etc.

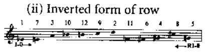
From these matrices a series was also derived for durations by reading each integer as numbers of demi-semiquavers. For example, at the very beginning Piano I plays the pitch classes of the original row, but with the durations of the final inverted, retrograded row (12, 11, 9, 10, 3, . . .)—see Figure 4b.

Furthermore, each statement of the row was assigned a particular dynamic and mode of attack determined by the matrices—Figure 4c shows the row of 12 dynamics and 10 modes of attack. The selection of dynamic and mode of attack is determined by reading diagonally across the matrices: the **P**-matrix for Piano I, the **I**-matrix for Piano II. Even the order in which the 48-note and 48-duration series are chosen is determined by the number matrices: for instance, the first twelve-note series in Piano I are those of the **P**-matrix but in the order of the numbers of the first row of the I-matrix (1, 7, 3, 10, . . .).

(a) (i) Prime or original form of row



(ii) Inverted form of row



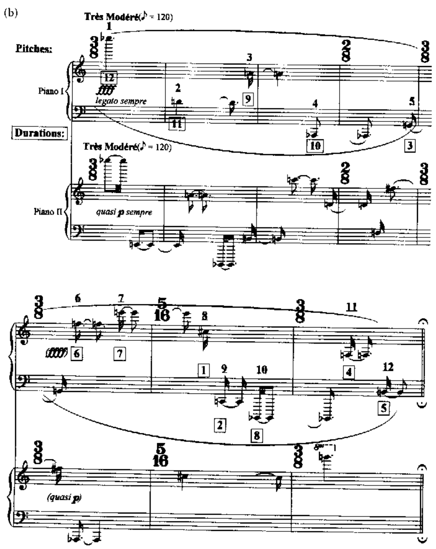
1	2	3	4	5	6	7	8	9	10	11	12
2	8	4	5	6	11	1	9	12	3	7	10
3	4	1	2	8	9	10	5	6	7	12	11
4	5	2	8	9	12	3	6	11	1	10	7
5	6	8	9	12	10	4	11	7	2	3	1
6	11	9	12	10	3	5	7	1	8	4	2
7	1	10	3	4	5	11	2	8	12	6	9
8	9	5	6	11	7	2	12	10	4	1	3
9	12	6	11	7	1	8	10	3	5	2	4
10	3	7	1	2	8	12	4	5	11	9	6
11	7	12	10	3	4	6	1	2	9	5	8
12	10	11	7	1	2	9	3	4	6	8	5

'P' MATRIX

1	7	3	10	12	9	2	11	6	4	8	5
7	11	10	12	9	8	1	6	5	3	2	4
3	10	1	7	11	6	4	12	9	2	5	8
10	12	7	11	6	5	3	9	8	1	4	2
12	9	11	6	5	4	10	8	2	7	3	1
9	8	6	5	4	3	12	2	1	11	10	7
2	1	4	3	10	12	8	7	11	5	9	6
11	6	12	9	8	2	7	5	4	10	1	3
6	5	9	8	2	1	11	4	3	12	7	10
4	3	2	1	7	11	5	10	12	8	6	9
8	2	5	4	3	10	9	1	7	6	12	11
5	4	8	2	1	7	6	3	10	9	11	12

'I' MATRIX

(b)



(c) DYNAMICS ROW

MODE OF ATTACK ROW

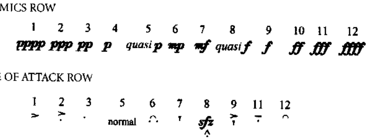


Figure 4. Boulez, *Structure Ia*.

Where does this leave the composer? What scope is there for him or her, in Schoenberg's words, to compose 'as freely as before'? Not too much, apparently. Though Boulez makes free choices regarding register, tempo, metre, and even the use of rests, his hands were tied by the system. The end result is so highly over-determined that it ends up sounding almost completely random: the differences are not readily discernible between *Structure Ia* and, say, Cage's near-contemporary *Music of changes*, where chance procedures of coin-tossing and use of the I-Ching were used to determine the various musical parameters. As an experiment in number made audible, Boulez's *Structures* are fascinating, but he was soon to admit that 'composition and organisation cannot be confused with falling into a maniacal inanity, undreamt of by Webern himself'. Whether or not *Structures* is maniacally inane is for the individual listener to decide.

Peter Maxwell Davies

The English composer, Peter Maxwell Davies, began his composing life as a follower of the thinking of Schoenberg and showed an early familiarity with serially derived techniques of composition. There is, as Paul Griffiths has pointed out, a kinship between the work of Maxwell Davies and Boulez of the mid-1950s 'in matters of rhythmic style, texture and serial handling'. Though their paths have subsequently gone in very different ways, there is a striking similarity in their attitude to number in generating musical material in some of their works. In particular, procedures in those works of Maxwell Davies of the 1970s which 'process' pitch and durational material through magic squares are not that dissimilar from some of Boulez's working in *Structures*.

Ave maris stella (1975) is one such example in which the Gregorian chant 'Ave Maris Stella' is, in Maxwell Davies's words, '"projected" through the magic square of the moon'. *A mirror of whitening light* (1976–7) is another. The title, according to the composer, refers to the alchemical process of purification or 'whitening', 'by which a base metal may be transformed into gold, and, by extension, to the purification of the human soul'. The 'agent' of this transformation is the spirit Mercury, represented here by the magic square of Mercury, and through which is projected the plainchant *Veni sancte spiritus*. The number 8, Davies tells us, 'governs the whole structure', and its source is the 8×8 'Magic square of Mercury' in Figure 5a, in which each row and column and each diagonal adds up to 260.

Figure 5b shows the way in which the plainchant is projected through the magic square. An 8-note 'summary' was derived from the beginning of the chant and consists of 8 different pitches, though it still maintains the profile of the original. An 8×8 matrix was then constructed in which each note of the summary was transposed, just like a tone row, to begin on each of its constituent notes, and each note was numbered consecutively from 1 to 64. The final stage was to map this matrix on to the magic square.

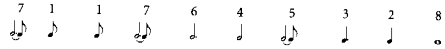
The composer then charted various courses through the matrix to generate pitch material: from top to bottom, left to right; from bottom to top, right to left; diagonally; in spirals; indeed, in any way he chose. Figure 5c shows how this is achieved at the opening of the work—in this case, top-bottom, left-right (C, A, B♭, F♯, D, D, . . .). It is just one way in which notes were generated in this piece, one aspect of the various transformations or 'whitenings' that the plainchant undergoes.

Durational lengths can also be determined by the Mercury matrix—Figure 5d shows one such instance. The pitches of the clarinet line were generated by starting at 'square 47' (see Figure 5b) and working backwards and upwards:

47 [B♭], 17 [F], 33 [F♯], 31 [D♯], 30 [E], . . .

The durations of the accompanying bassoon line use the same numerical sequence from the magic square, but here all the numbers were converted so that they lie within the range 1 to 8, by reducing them modulo 8 (for an explanation of modular arithmetic, see Chapter 9). This new but related numerical array was then taken to represent numbers of quaver beats and is stated in the opposite direction from the pitch 'row':

clarinet 'pitch row'	47	17	33	31	30	36	37	27	26	40
bassoon 'duration row'	7	1	1	7	6	4	5	3	2	8



The bassoon's pitches, incidentally, were generated by a left-right reading of the Mercury matrix starting, as at the beginning of the work, in the top-left corner.

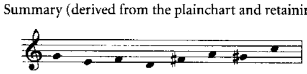
Can any of this be heard? Maxwell Davies has great faith in his listeners: these 'sequences of pitches and rhythmic lengths . . . [are] easily memorable once the "key" to the square has been found', he claims. No doubt he would argue that the 'logic' given to the various transformations by the magic square is, at the very least, subconsciously perceived. I have my doubts. What one *hears* is a piece of music, clearly structured with a focal 'key' centre of C, and not just a mathematical game made audible. However, the numbers were vital to the compositional process, as they were a means of providing the composer with his working material. To misappropriate Schoenberg, one has to follow the magic square; but, nevertheless, one composes as before. As Maxwell Davies himself has said in the context of his later *Second symphony*, magic squares 'are a gift to composers if used very simply as an architectural module'.

(a) Magic square of Mercury

8	58	59	5	4	62	63	1
49	15	14	52	53	11	10	56
41	23	22	44	45	19	18	48
32	34	35	29	28	38	39	25
40	26	27	37	36	30	31	33
17	47	46	20	21	43	42	24
9	55	54	12	13	51	50	16
64	2	3	61	60	6	7	57

(b)

Summary (derived from the plainchart and retaining its profile)



G 1	E 2	F 3	D 4	F♯ 5	A 6	G♯ 7	C 8
E 9	C♯ 10	D 11	B 12	D♯ 13	F♯ 14	F 15	A 16
F 17	D 18	E♭ 19	C 20	E 21	G 22	F♯ 23	A♯ 24
D 25	B 26	C 27	A 28	C♯ 29	E 30	D♯ 31	G 32
F♯ 33	D♯ 34	E 35	D♭ 36	F 37	A♭ 38	G 39	B 40
A 41	F♯ 42	G 43	E 44	G♯ 45	B 46	B♭ 47	D 48
G♯ 49	F 50	F♯ 51	E♭ 52	G 53	B♭ 54	A 55	C♯ 56
C 57	A 58	B♭ 59	G 60	B 61	D 62	C♯ 63	F 64

8 × 8 matrix

C 8	A 58	B♭ 59	F♯ 5	D 4	D 62	C♯ 63	G 1
G♯ 49	F 15	F♯ 14	E♭ 52	G 53	D 17	C♯ 10	C♯ 56
A 47	F♯ 23	G 22	E 44	G♯ (D) 45	E♭ 19	D 18	D (G♯) 48
G 32	D♯ 34	E 35	C♯ 29	A (D) 28	A♭ 38	G 39	D (A) 25
B 40	B 26	C 27	F 37	D♭ 36	E 30	D♯ 31	F♯ 33
F 17	B♭ 47	B 46	C 20	E 21	G 43	F♯ 42	A♯ 24
E 9	A 55	B♭ 54	B 12	D♯ 13	F♯ 51	F 50	A 16
F 64	E 2	F 3	B 61	G 60	A 6	G♯ 7	C 57

Magic square of Mercury

Figure 5. *Peter Maxwell Davies, A mirror of whitening light.*

Figure 5. Peter Maxwell Davies, *A mirror of whitening light*.

Figure 5. Peter Maxwell Davies, *A mirror of whitening light*.

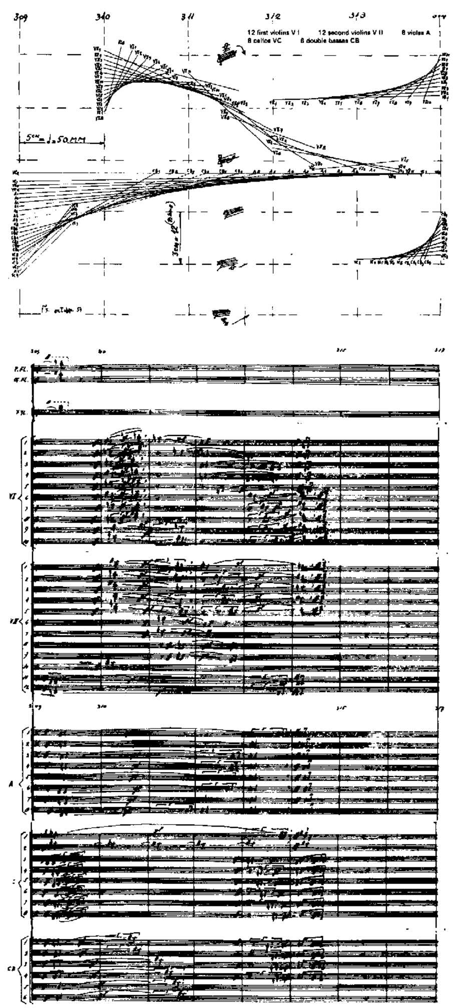


Figure 6. Xenakis, *Metastasis*.

Iannis Xenakis

Such a sentiment was also close to the heart of another composer for whom an understanding of mathematics and architecture were fundamental. Iannis Xenakis was born of Greek parentage and educated in Greece; ancient Greek culture—be it drama, architecture, philosophy or mathematics—continued to have a profound influence on his thought.

Xenakis's early education was principally as an engineer, and when he moved to Paris in 1947 he not only studied with the composers Honegger, Milhaud and Messiaen, but also met the architect Le Corbusier with whom he was to collaborate on a number of important projects. Their most celebrated undertaking was for the Philips Pavilion (Figure 7) at the 1958 Brussels World Fair for which, in just a few days, Xenakis sketched the basic structure using conoids and hyperbolic paraboloids. As Xenakis later observed:

I discovered on coming into contact with Le Corbusier that the problems of architecture, as he formulated them, were the same as I encountered in music.

And elsewhere:

With Le Corbusier I discovered architecture; being an engineer I could do calculations as well, so I was doing both. This is quite rare in the domain of architecture and music. Everything started coming together and I also asked musical and philosophical questions.

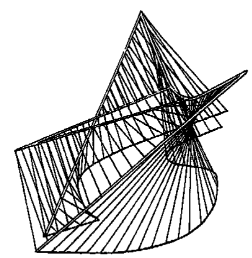


Figure 7. First model of the Philips pavilion. Its structure is generated by straight lines.

It would seem, then, that for Xenakis music and architecture were concerned with the same issues: in architecture his ideas were articulated in space; in music they were articulated in time. Furthermore, mathematical models underpinned the development of his ideas in both realms.

His first acknowledged composition, *Metastasis* ('transformations') of 1953–4, clearly exemplifies these concerns. The structure of the curved surfaces of the Philips Pavilion was generated by straight lines; *Metastasis* had already demonstrated, as Xenakis put it, that it was 'possible to produce ruled surfaces by drawing the glissandi as straight lines'. Music and architecture here found an intimate connection, as we can see if we compare Xenakis's graph plotting the paths of a section of glissandi with the same passage in the score—see Figure 6.

Metastasis shows Xenakis exploring architecturally derived notions of mass and ruled surface, and a concern to represent 'sound events made out of a large number of individual sounds [which] are not separately perceptible, . . . [to] reunite them again . . . [so that] a new sound is formed which may be perceived in its entirety'. In *Metastasis* one is not aware of individual sounds but of a new mass of sounds and timbres. The means by which he achieved this were derived from *The modulor* of Le Corbusier: pitches (based on twelve-note rows) were assigned a series of durations based on the Fibonacci sequence, along with a range of timbres. The way in which this material was processed *became* the form of the piece.

Xenakis subsequently developed these ideas in a much more comprehensive way, using many mathematical models as well as computers to assist him in his pre-compositional calculations. He soon became interested in probability theory as a way of handling mass sound phenomena, and from this grew what he described as 'large number' or 'stochastic' music, where the operation of individual elements is unpredictable but the shape of the whole can be determined. For example, *Pithoprakta*, the next work after *Metastasis*, drew (the composer claimed) on Maxwell–Boltzmann's kinetic theory of gases; *Achorripsis* employed Poisson's law; and *Duel* and *Stratégie* used game theory—each work employed two conductors who 'compete' with one another. More recently, Xenakis developed what he called 'symbolic music' which drew on principles of symbolic logic. Paul Griffiths has observed that 'Xenakis's symbolic music has . . . the nature of a translation into sound of theorems of set theory', first evident in *Herma* for piano of 1960–1.

This may suggest that Xenakis's music is completely abstract and sterile. Not at all. His music, like the man, is all too human and he always asserted the primacy of music over mathematics—music, he believed, is never reducible to mathematics, even though they have many elements in common. Xenakis was a philosopher who expressed his ideas primarily in music, but who was constantly searching for profound fundamental principles that underlie all thought. As another commentator has put it, 'he gives us something only an artist can give—a dynamic picture of the universe informed by the science of today'.

Although Xenakis's use of a variety of mathematical models may have been undertaken in a more consistent and thoroughgoing manner than almost any other composer, it does not make his music any less exciting, challenging, creative—or even valid—than music composed in a different age or by different means. Mathematics is a means to an end, not the end in itself. Composers today are as aware as have been thinkers of the past that music is inherently mathematical, but this does not mean to say that it *is* mathematics. Composing with numbers is not an admission of compositional failure, a substitute for 'inspiration' or 'musicality', whatever those concepts may mean. Composers have composed with numbers as one way of generating new musical ideas, as a means of stimulating their creativity, in answer to the fundamental questions posed for all artists of the last century. In Xenakis's words, this represents:

> the effort to make "art" while "geometrizing", that is, by giving it a reasoned support less perishable than the impulse of the moment, and hence more serious, more worthy of the fierce fight which the human intelligence wages in all the other domains.