Engineering and Science Mathematics 2B

Homework 1

due February 11, 2004

Normal questions and advanced questions (A) are worth 5 points; easy questions (E) are worth 4 points. Complete either the easy, or the advanced version, not both.

1. Show that $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b} \cdot (\boldsymbol{c} \times \boldsymbol{a})$.

2. (E) Find the distance between the point $(5, 12, -13)$ and the plane with equation $3x + 4y + 5z = 12$.

   (A) Show that the distance of the point (x_0, y_0, z_0) to the plane $ax + by + cz = d$ is given by
   $$D = \frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}\,.$$

3. (E) Find the distance between the point $\boldsymbol{p} = (1, 2, 3)$ and the line
   $$\boldsymbol{x} = \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}\,.$$

   (A) Show that, as an alternative to the formula given in class, the distance between a point $\boldsymbol{p}$ and the line $\boldsymbol{x} = \boldsymbol{a} + \lambda \boldsymbol{v}$ is given by
   $$d = |\boldsymbol{w} - \boldsymbol{w} \cdot \hat{\boldsymbol{v}}\, \hat{\boldsymbol{v}}|\,,$$
   where $\boldsymbol{w} = \boldsymbol{a} - \boldsymbol{p}$, and $\hat{\boldsymbol{v}}$ is the unit vector in the direction of $\boldsymbol{v}$.

4. Find an equation for the plane that contains the point $\boldsymbol{p} = (2, 4, 6)$ and the line
   $$\boldsymbol{x} = \begin{pmatrix} 7 \\ 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}\,.$$

5. (E) Find the angle between the vectors $(3, -4, 0)$ and $(-2, 1, 0)$, and find a vector that is perpendicular to both.

   (A) Prove, by writing out in component form or by following the suggestion in Edwards & Penney, p. 733), that
   $$(\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c} = (\boldsymbol{a} \cdot \boldsymbol{c})\, \boldsymbol{b} - (\boldsymbol{b} \cdot \boldsymbol{c})\, \boldsymbol{a}\,.$$

6. Let $z = 3 + 4i$ and $w = -5$. Sketch the following quantities in the complex plane: z^*, $z + w$, $z - w$, zw, z/w. (This is sometimes called an Argand diagram plot.)

7. Simplify the following expressions:

   (a) $\mathrm{Re}\,\dfrac{1+i}{1-i}$

   (b) $\mathrm{Im}(\exp 2iz)$

   (c) $\ln i$