Engineering and Science Mathematics 2B

Review for the Final Exam

May 21, 2004, 8:00–10:00, Naber Lecture Hall

1. Solve a system of linear equations, matrix inversion.

   Study problems: There are plenty of practice problems on homework, past exams, and in the book.

2. Concept of vector space, linear independence, basis; linear transformations: definition, representation by a matrix; change of basis.

   Study problem: See, in particular, question 6 on Midterm 1 from Spring 2003.

3. Eigenvalues and eigenvectors, in particular for Hermitian matrices; determinants; diagonalization; inner products.

   Study problems: See, in particular, questions 1 and 2 of Midterm 2 from Spring 2003, and questions 2 and 6 from Homework 5.

4. Fourier Series: compute the series, properties. Concentrate on the complex Fourier series!

   Study problems: There are plenty of practice problems on homework, past exams, and in the book.

5. Fourier transform: Compute the transform, inverse Fourier transform, properties, Fourier transform of the convolution of two functions, Parseval theorem.

   Study problem: Recall that the convolution of two functions f and g is defined by

   $$(f * g)(x) = \int_{-\infty}^{\infty} f(y)\, g(x-y)\, \mathrm{d}y\,.$$

   Show that $\mathcal{F}(f * g) = \sqrt{2\pi}\, \tilde{f}\, \tilde{g}$.

6. Delta function: Definition, representations; application of the delta function when computing the probability distribution function for functions of a continuous random variable.

   Study problem: The velocity of a molecule in a one-dimensional gas is modeled by a continuous random variable V whose probability distribution function is normal distribution with mean 0 and variance σ^2,

   $$f(v) = \frac{1}{\sqrt{2\pi}\sigma}\, \mathrm{e}^{-\frac{v^2}{2\sigma^2}}\,.$$

   The energy of each molecule is the random variable $E = V^2$. Compute the probability distribution function for E.

(For physicists: $\sigma^2 = kT/m$, where k is the Boltzmann constant, m the molecular mass, and T the temperature. In physical units, we have to write $E = m\,V^2$. For background information, see `http://hyperphysics.phy-astr.gsu.edu/hbase/kinetic/kintem.html`.)

7. Probability: Outcomes, events, sample spaces, definition of probability, conditional probability, Bayes' rule.

   Study problem: A machine sorts potatoes into small, medium and large ones, but it makes mistakes. Suppose that the probability that a random potato will end up in a specified category is according to the following table.

	categorized as		
potato of size	small	medium	large
small	0.8	0.15	0.05
medium	0.2	0.7	0.1
large	0.05	0.1	0.85

   Suppose further that 30 % of the potatoes are small, 45 % are medium and 25 % are large. What are the percentages of small, medium and large potatoes in each of the three piles the machine produces?

8. Permutations and Combinations; their use for the computation of probabilities; expected value and variance; binomial, Poisson, and Gaussian distribution; know how to compute the mean and variance using the moment generating function.

   Study problem: In a game you throw a pair of dice. If the sum of the values equals 12 you win 5 euros, if it equals 11 you win 1 euro; for all other outcomes you won't get anything.

   (a) Calculate the probability of winning exactly 6 euros after 6 games.

   (b) Compute the expected value of the pay-out per game.

9. Properties of random variables, functions of random variables.

   Questions to ask: If X and Y are random variables, determine if, or under what conditions, the following statements are true: $E[XY] = E[X]\,E[Y]$, $\mathrm{Var}[XY] = \mathrm{Var}[X]\ \mathrm{Var}[Y]$, $E[X+Y] = E[X] + E[Y]$, $\mathrm{Var}[X+Y] = \mathrm{Var}[X] + \mathrm{Var}[Y]$?

Notes:

- No calculators.
- Many computations will involve complex numbers. If you have difficulties with complex numbers, you should practice manipulating them.
- ESM 2A homework sheets for Linear Algebra and Probability are on the web and a good source for practice problems.

  `http://math.iu-bremen.de/stoll/teaching/ESM2A-2003-Spring/schedule.html`
- The following topics should also be revised, as they may be required as part of some question: Equations for lines and planes; distance of a point to a line or plane; distance between two lines; Orthonormal bases, Gram-Schmidt orthnormalization, Hermitian Matrices and operators.