Engineering and Science Mathematics 2B

Homework 8

due May 3, 2003, before 22:00

Normal questions and advanced questions (A) are worth 5 points; easy questions (E) are worth 4 points. Complete either the easy, or the advanced version, not both.

1. (E) Determine whether the following relationships are valid. For those that are, prove the relationship using de Morgan's law.
   (a) $\overline{(\overline{X} \cup Y)} = X \cap \overline{Y}$
   (b) $(X \cup Y) \cap Z = (X \cup Z) \cap Y$
   (c) $X \cup \overline{(Y \cap Z)} = (X \cup \overline{Y}) \cup \overline{Z}$

   (A) Given that events X, Y, and Z satisfy

   $$(X \cap Y) \cup (Z \cap X) \cup \overline{(\overline{X} \cup \overline{Y})} = \overline{(Z \cup \overline{Y})} \cup \left[\left(\overline{(\overline{Z} \cup \overline{X})} \cup (\overline{X} \cap Z)\right) \cap Y\right],$$

   prove that $X \supset Y$ and that $Z \cap X \subset Y$.

2. Let A and B be two mutually exclusive events. Suppose $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$. Compute the probabilities $P(A|B)$, $P(B|A)$, $P(A \cup B)$, $P(A \cap B)$, $P(A - B)$, and $P(B - A)$.

3. Gamblers A and B each roll a fair six-faced die, and B wins if his score is strictly greater than A's. Show that the odds are 7 to 5 in A's favor.

4. You witness a night-time hit-and-run accident involving a taxi in a city in which 9 out of 10 taxis are green, and all other taxis are blue. Discrimination between the two colors is 80% reliable under the dim lighting conditions at the scene. You are sure that the taxi was blue. What is the most likely color of the taxi, and what is the probability that your impression is correct?

5. *The Monty Hall Problem*

   Suppose you are on a game show, and you are given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors?

   Explain the answer using the notion of conditional probability.

6. Bayesylvania is awash with natives not all of whom can be trusted to tell the truth. A traveler finds himself at a T-junction in an area populated by the Asciis and Bisciis in the ratio 11 to 5. The Biscii always lie but the Ascii tell the truth three quarters of the time, giving independent answers to all questions, even to immediately repeated ones.

   (E) The traveler asks one particular native whether he should go to the left or to the right to reach the local village. He is told "left". Should he take this advice, and, if he does, what are his chances of reaching the village?

   (A) The traveler asks one particular native three times whether he should go to the left or to the right to reach the local village. Each time he receives the same answer "left". What should the traveler do now? Have his chances of finding the village been altered by asking the question three times?