Problem 3 of homework 9:

(E)

The random variable G_x can take the values x^2 (in case we win) and $-x$ (in case we lose). Since we win with probability p_x we lose with probability $1 - p_x$:

$$E[G_x] = p_x\, x^2 + (1 - p_x)\,(-x) = p_x\,(x^2 + x) - x.$$

$$E[G_2] = -\frac{13}{8}, \quad E[G_3] = -\frac{3}{2}, \quad E[G_4] = -\frac{1}{4}, \quad E[G_5] = \frac{5}{2},$$
$$E[G_6] = \frac{15}{8}, \quad E[G_7] = 0, \quad E[G_8] = -\frac{7}{2}.$$

(A)

If we always guess the number obtained in the previous—statistically independent—trial, then this number plays the role of a random variable X, and the expected payout for a particular value of the random variable as has been calculated in (E) is now a *function of the random variable* X. The expected payout is therefore

$$\begin{aligned} E[E[G_X]] &= \sum_{x=2}^{8} p_x\, E[G_x] \\ &= \tfrac{1}{16}\,\tfrac{-13}{8} + \tfrac{2}{16}\,\tfrac{-3}{2} + \tfrac{3}{16}\,\tfrac{-1}{4} + \tfrac{4}{16}\,\tfrac{5}{2} + \tfrac{3}{16}\,\tfrac{15}{8} + \tfrac{2}{16}\,0 + \tfrac{1}{16}\,\tfrac{-7}{2} \\ &= \tfrac{27}{64}. \end{aligned}$$